Estimates of Extremes in the Best of All Possible Worlds

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Outline

1. Prediction in Hydrology
    - Can it be done?
    - How to see what happens if we do
2. Indicators of uncertainty
    - Formalizing the problem
    - Numerical experiment
    - Bayesian
    - "Frequentist"

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Reasons why it should not be done.

For more reasons read Klemeš

- Systems are not stationary.
- Time series are not generated from independent identically distributed random variables.
- Not enough data.

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Other fields where predictions are made.

- Econometrics.
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Reasons why perhaps it can be done.

- There is theory on extremes for non-stationary time series
- There is theory on extremes for non-iid time series
- There is theory on extremes for time series with correlation

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Procedures

- Numerical experiments
- Frequentist approach to quantile as a statistic
- Bayesian approach to quantile as a statistic

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Notation

- Random variable representing the event: X
- Random vector representing a set of n observations $\vec{Y}$

pdf f_X

cdf F_X

quantile function $Q_X = F_X^{-1}$

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A question of Definition

- Suppose cdf, pdf, quantile function are parameter dependent
- For known parameter $\theta = \theta_0$
- $\Pr\left(X \leq Q_X\left(p_0, \theta_0\right) \mid \theta = \theta_0\right) = F_X\left(Q_X\left(p_0, \theta_0\right), \theta_0\right) = p_0$
- But if we *estimate* the parameter then in general
- $\Pr\left(X \leq Q_X\left(p_0, \hat{\theta}\left(\vec{y}\right)\right) \mid \theta = \theta_0\right) \neq p_0$

Gumbel

$$F_{\mathrm{G}}(x \mid \langle \xi, \zeta \rangle) = \exp\left(-\exp\left(-\frac{x-\xi}{\zeta}\right)\right)$$

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Numerical experiment

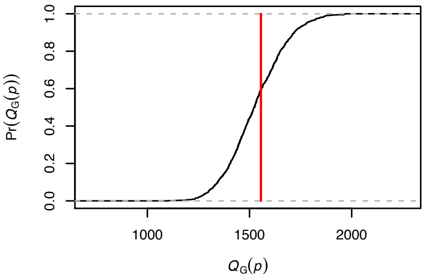

Figure : ecdf of quantile for $p = 0.99$ based on ML parameters, sample size $m = 20$

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Quantile distribution

- $\Pr\left(Q_X\left(p_0 \mid \vec{\Theta}\right) \leq t\right)$ is the distribution of the quantile function
- For a given posterior distribution of $\vec{\Theta}$.

Outline

Upper bound with given probability of exceedance.

- A function $b_{\text{ub}}(p, \vec{y})$
- such that
- we have $\Pr\left(X \leq b_{\text{ub}}\left(p, \vec{Y}\right)\right) = p$

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Upper bound on 1:N event size with a probability of exceedance.

- A function $b_{\text{ub}}(\alpha, p, \vec{y})$
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Upper bound on 1:100 event, 10% probability of exceedance, $m = 20$

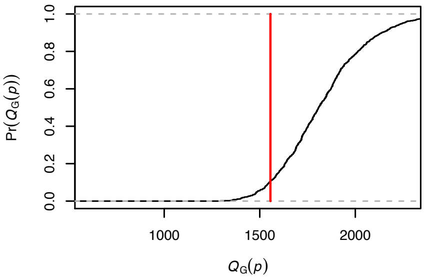
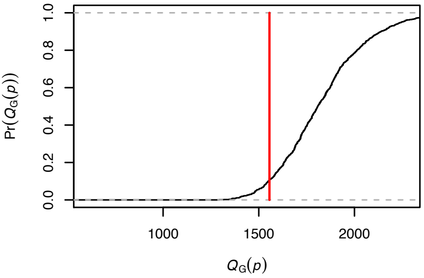
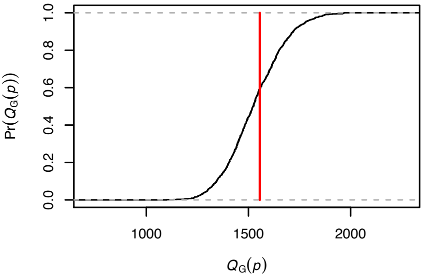


Figure : ecdf of upper bound for quantile estimates for $m = 20$

Comparison


0.0
0.2
0.4
0.6
0.8
1.0
Pr($Q_G(p)$)
1000
1500
2000
$Q_G(p)$


0.0
0.2
0.4
0.6
0.8
1.0
Pr($Q_G(p)$)
1000
1500
2000
$Q_G(p)$

Upper bound on 1:100 event, 10% probability of exceedance, $m = 100$.

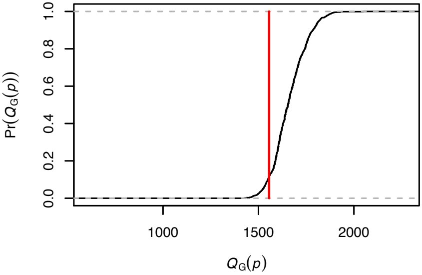


Figure : ecdf of upper bound for quantile estimates for $m = 100$

Summary

- There are ways to display uncertainty.
- For small samples the prediction uncertainty can be large.
- Both "frequentist" and "Bayesian" techniques can be applied.

For Further Reading I

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