A COMPARISON OF FITTING METHODS AND TESTS FOR SEVERAL DISTRIBUTIONS ON HYDROLOGICAL DATA

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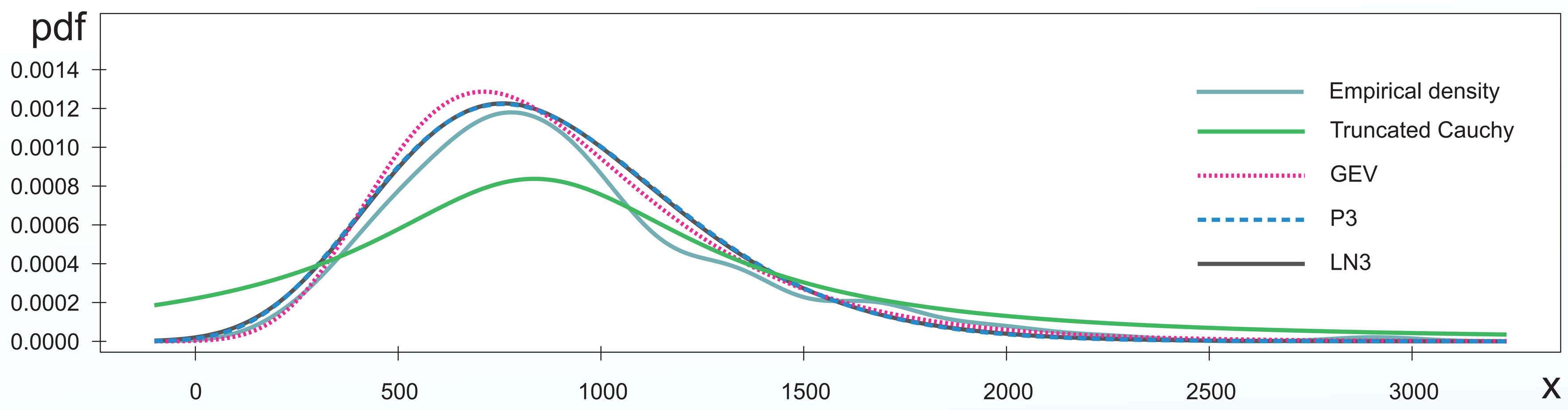
Introduction

We considered two applications of distributions in hydrology: extrapolation, used when predicting low flow or high flow extremes, and interpolation, used in the context of or rainfall intensity-duration-frequency curves or synthetic unit hydrographs. We used prediction of a 1:100 event from a 50 point sample as a model for the extrapolation case and we used determination of the first, second and third quartile as a model for the interpolation case.

We considered the Generalized Extreme Value (GEV) distribution, a Pearson III (P3) distribution and a Three parameter lognormal (LN3). We fitted with the method of moments (MoM), with L-moments and with Maximum Likelihood (ML). We applied Kolmogorov-Smirnov (KS), Cramér-von Mises (W2) and Anderson-Darling (AD2) tests to the resulting fit. The p-value was 0.1 and its corresponding value for the statistic was determined from Monte Carlo simulations.

We looked for relations between the original sample, the distribution, the fitting method, the test and the purpose of the fit.



The probability density function (pdf) of GEV, P3, LN3 and the empirical pdf for the sample as constructed by R. Also shown is the pdf for the truncated Cauchy distribution. The GEV, P3, LN3 and the empirical distribution have their first quartile at 633.5, median at 843 and third quartile at 1084 (for the empirical distribution the quantiles were calculated with the R quantile routine).

Visualization of Results

The Tukey plots show first, second and third quartile as a box and whiskers to the leftmost and rightmost data points within a range of 1.5 times the inter quartile distance. Points outside that range are shown as circles or, when they lie too far outside the range, with annotations in the plots.

Correspondence Between Data and Distribution

For the quartiles none of the distributions provides a clearly better fit, but without a test LN3 has a larger spread due to points far from the median. For the extreme value P3 seems to do slightly better than the others with respect to bias, except for one sample where it puts the extreme value at 6300.

Relationships

Fitting Method and Suitability of the Resulting Fit (Extreme Value)

The method of moments (MoM) did well for GEV, LN3 and P3 without a test, but seemed to underestimate the value when combined with a test. For GEV with a test maximum likelihood (ML) had a smaller bias than L-moments. For LN3 with a test ML and MoLM did equally well, perhaps with a slight advantage for ML. For P3 both ML and L-moments did well, but ML seemed to do slightly better.

Fitting Method and Suitability of the Resulting Fit (Quartiles)

For the quartiles there seems to be no clear winner for GEV, although perhaps ML has slightly less bias for the third quartile than MoLM, which in turn does slightly better than MoM. For LN3 it is hard to point to a clear favorite for the three quartiles together. The same holds for P3.

Can Goodness of Fit Tests Improve the Suitability of a Fit for a Particular Purpose?

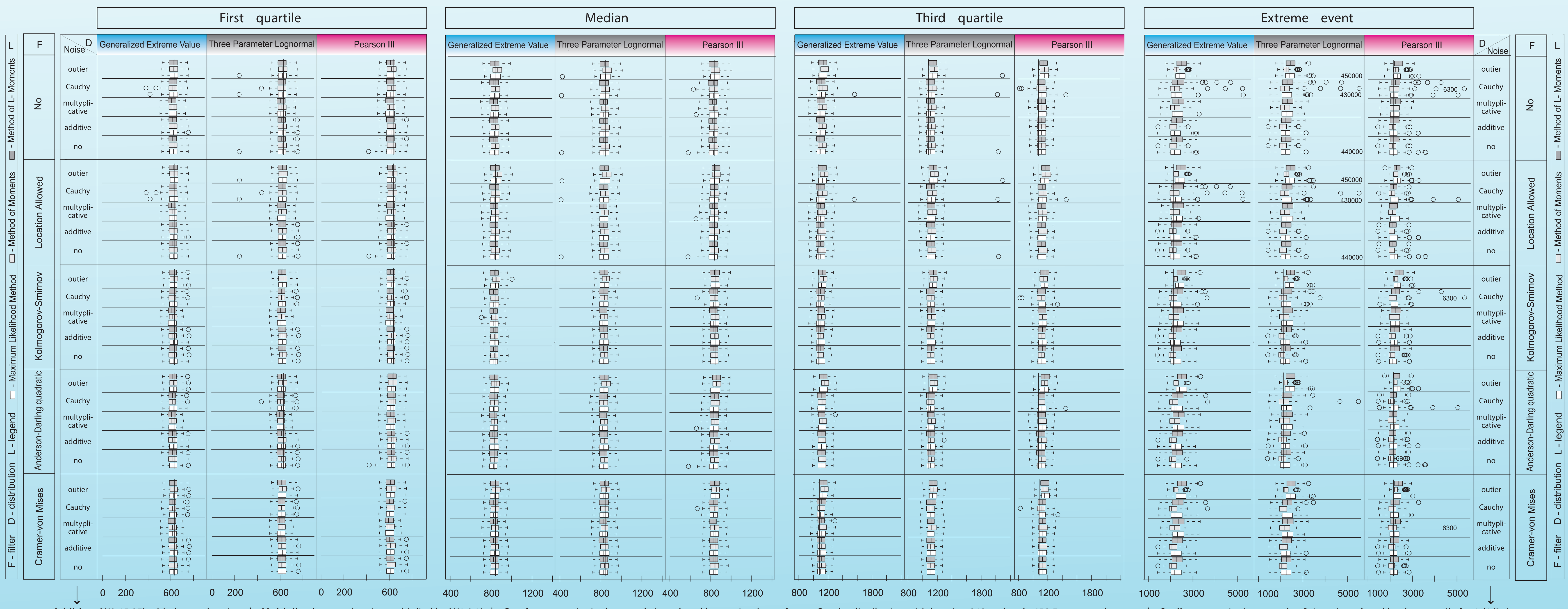
Kolmogorov-Smirnov, Cramer-von Mises and Anderson-Darling all remove some of the extreme values for the quartiles. For the extreme value the effect is most pronounced for GEV. The Anderson-Darling test is best as a filter when looking at the extreme value for P3.

Success rate

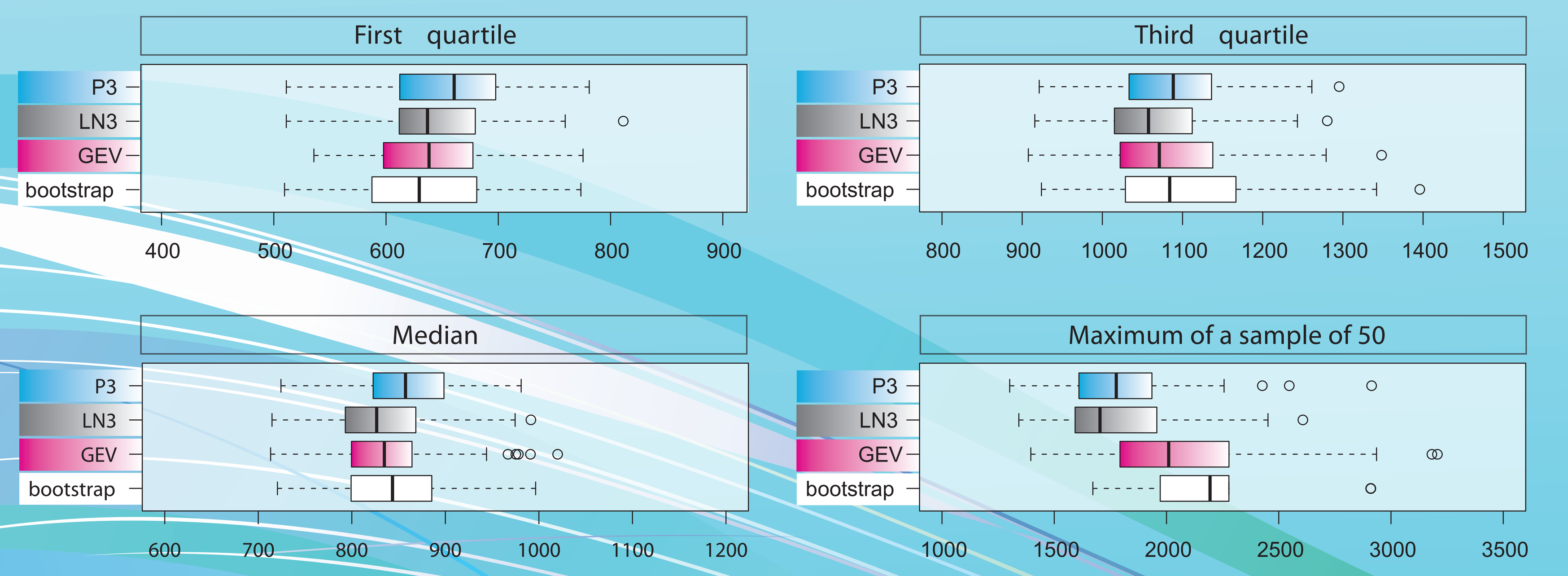
For LN3 and P3 all fitting methods succeeded for all samples. For GEV the MoM and MoLM methods succeeded for all samples, ML succeeded for 96 out of 100 clean samples and for between 92 and 96 out of 100 perturbed samples. There are four kinds of failure:

1. Failure of the numerical algorithm to converge,
2. Failure of the assumptions implicit in the choice of distribution (for instance: an attempt to fit a distribution with positive skew or L-skew to a sample with negative skew with MoM or MoLM),
3. A fit that places some sample points out side the range of possible results (for example: LN3 with location to the right of the smallest sample point),
4. Convergence of the algorithm to what seems to be a wrong answer (For instance the P3 extreme value at 6300).

Interesting question: how should a fitting routine react when these failures occur?



Additive: N(0,45.05) added to each point | **Multiplicative:** each point multiplied by N(1,0.1) | **Cauchy:** one point in the sample is replaced by a point drawn from a Cauchy distribution with location 843 and scale 450.5, truncated at zero | **Outlier:** one point in a sample of size n is replaced by the quantile for 1-(1/2n)



Effectiveness of bootstrapping

To simulate multiple samples from a physical system we used a 175 point time series as a basis for a bootstrapping procedure. To examine the properties of the resulting samples we constructed GEV, LN3 and P3 distributions with the same quartiles as the time series data. We then took 100 samples of 50 points each from these distributions and made Tukey plots of the sample quartiles and sample maxima. We did the same for the bootstrapped samples. The results show that the quartiles seem to be fairly realistic.

As to the maximum, the empirical distribution puts the 1 in 50 event at 1986.56 and the empirical 1 in 100 event at 2153.68 (for the empirical distribution the quantiles were calculated with the R quantile routine). We gave the distributions matching quartiles, so the clear differences between the distributions seen here are not unexpected. The spread of the bootstrap sample results is comparable to that of the other samples, but the upper quartile seems a bit narrow.

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