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Stochastics in hydrometeorological processes: from point to global spatial scales and from minute to climatic time scales

Stochastic Index estimation of flow duration curves on ephemeral basins

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Abstract:

Availability of tools to simulate discharges regime, in particular low flow regime is a fundamental topic in the planning and management of water resources. The correct representation of a river frequency regime is in fact really useful in different hydrological applications, as analysis of hydro-electrical feasibility of a river, in the management of water resources and in environmental planning. In this field a fundamental problem is the flush of waste water in rivers that has no flow for percentage of time bigger than a fixed value.
In this situation it is not useful get an only index, but it is necessary to use flow duration curves (FDC). This curve describes river discharges versus the percentage of time that these are exceeded.
Therefore this work combines the stochastic index model, that enables to reproduce the period of record FDC of a river without regard persistency and seasonality of the series, for calculate conditional probability F(y|Y>0), with the theory of total probability, that allows to evaluate the percentage of time the river is dry. This procedure is applied to a nested catchment in the Lazio region.

1. Flow Duration Curves

The FDC for a series of daily flows is the complement of the cumulative distribution function of the daily streamflows based on the complete record of flows.
Different approaches can be used to construct FDC (non parametric approaches for estimate FDC are in Vogel and Fennessey [1994]). A non parametric approach to constructing an FDC can be used:
- rank the observed streamflows in ascending order;
- plot each ordered observation versus its corresponding duration or exceedance probability. The duration is often expressed as a percentage, and it coincides with an estimate of the exceedance probability, ε_i, of the *i*-th observation in the ordered sample.

ε_i can be estimated using a Weibull plotting position. Then the duration D_i is:

$D_i = 100(\varepsilon_i) = 100\cdot\left(1-\frac{i}{n+1}\right)$, for $i = 1,2,3,\ldots, n$

where *n* is the length of the sample.

2. Stochastic Index Flow Model of Flow Duration Curves

The approach for sthocastic modeling daily stramflows is similar to index flood approach.
The approach assumes that the daily streamflow X can be found multiplying an index flow equal to the annual flow (AF) and a dimensionless daily streamflow X',

$$X = AF\cdot X'$$

AF describes the long-term climatic regime for a given basin. It is correlated with mean annual precipitation. The probability density function (pdf), $f_{X'}$, of standardized flows is correlated with the hydrologic regime, and geomorphological characteristics of basin.

3. Period of Record Flow Duration Curves (FDC)

The FDC based on the complete period of record of flows is the complement of the cumulative distribution function (cdf) of X, F_X given by

$$F_x(x) = P\{X\le x\} = \int_{xl}^{x} f_x(u)du = P\{AF\cdot X'\le x\} = \int_{\Omega X'}\int_{af}^{x/z} f_{AF,X'}(v,z)dvdz$$

where Ω_Y indicates the domain of a given random variable Y, f_X is the pdf of X, f_{AF},X' represents the joint probability distribution of AF and X0, and x_l and af_l are the lower bounds of $\Omega_{X'}$ and Ω_{AF}, respectively. If AF and X' are assumed to be independent, then $f_{AF,X'}$ equals the product of the two marginal distributions, and the equation becomes:

$$F_X = \int_{\Omega X'} f_{X'}(z)\cdot F_{AF}(x/z)dz$$

where F_{AF} is the cdf of AF and $f_{X'}$ is the pdf of X'. The last equation can be solved analytically or numerically, provided expressions for F_{AF} and $f_{X'}$. The desired FDC can then be constructed by plotting the variable X versus the duration, equal to $100(1-F_X)$

4. Theorem of total probability for random variables with zero value and use for estimate of FDC

The theorem of total probability can be used to determine the probability of occurrence of a non-zero event, given that a zero event has already occurred (Jennings & Benson, 1969).
The theorem is given by:

$$Pr(X > x) = Pr(X > x \mid X = 0)Pr(X = 0) + Pr(X > x \mid X \neq 0)Pr(X \neq 0)$$

But Pr(X >x | X = 0) is zero, as river flows cannot be negative, then the relationship reduces to:

$$Pr(X > x) = Pr(X > x \mid X \neq 0)\, Pr(X \neq 0)$$

If this relationship is written in the form of the cumulative probability distributions:

$$Pr(X<x) = pdry + pnz\, Pr(X < x \mid X \neq 0)$$

pnz = the proportion of time that the river is flowing, *(i.e. Pr(X ≠ 0)). Pnz can be estimated using plotting position formula.*

Pr(X < x|X ≠ 0) = $\int_{\Omega X'} f_{Xnz'}(z)\cdot F_{AFnz}(x/z)dz$

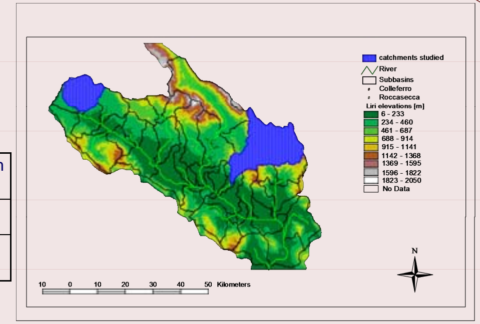
Calculation of the conditional distribution F_{AFnz}=Pr(AF<af |AF ≠0) and $F_{X'NZ}$=Pr(X'<x'|X' ≠0) with positive values of the series, can be done with a fitting procedure. Empirical frequency distribution conditioned on Y>0 can be calculated on k non zero values using the weibull plotting postion: $\left(\frac{i}{k+1}\right)$
Then general formulation of Stochastic index flow model for the use with zero values is:

$$Pr(X<x) = pdry + pnz\cdot \int_{\Omega Xnz'} f_{Xnz'}(z)\cdot F_{AFnz}(x/z)dz$$

5. Case Study

The method is applied to 2 sub-catchments of Liri-Garigliano river. Liri-Garigliano basin area is of 4900 km². The river net is controlled by apenninic and anti-apenninic mountains. The catchment has a carbonatic layer that causes high infiltration and low runoff.

Stations	Area [km²]	H min [m]	H max [m]	H mean [m]	River length [km]
Roccasecca	343.4	124	2050	793.8	33
Colleferro	138.5	211	975	362	13



6. Results

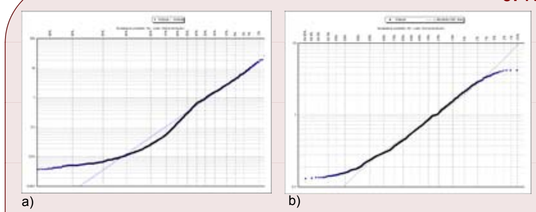


Fig.1 Fitted Conditional Frequency Distribution $F_{X'NZ}$=Pr(X'<x'|X' ≠0) for a) Roccasecca station, b) Colleferro Station

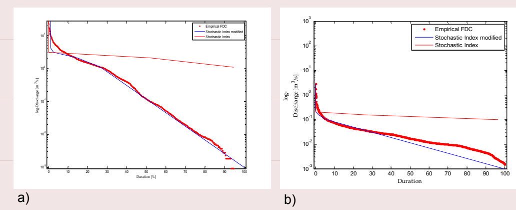


Fig.1 Fitted FDC Pr(X>x) for a) Roccasecca station, b) Colleferro Station.

Stations	RMSE SI [m³/s]	RMSE SI mod [m³/s]	Nash-Sutcliff SI	Nash-Sutcliff SI mod
Roccasecca	14.9	5.2	0.95	0.97
Colleferro	12.2	7	0.06	0.47

7. Conclusions

Intermittent rivers, arid and semiarid regions, have samples with zero events. This create a discontinuous frequency distribution, and creates problems on calculation of flow duration curves.

The method here developed joins stochastic index theory for calculation of conditioned frequency distribution, with theorem of total probability for calculation of probability of non zero events.

The model gives good and encouraging results, as it is possible to see from table of errors and figures.

More tests are needed in different stations and also Annual FDC must be calculated in intermittent rivers for have information on mean flow duration curves.

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