Properties and performance of a copula-based design storm generator

S. Vandenberghe[1] N.E.C. Verhoest[1] B. De Baets[2]

[1]Laboratory of Hydrology and Water Management (Ghent)

[2]Department of Applied Mathematics, Biometrics and Process Control (Ghent)

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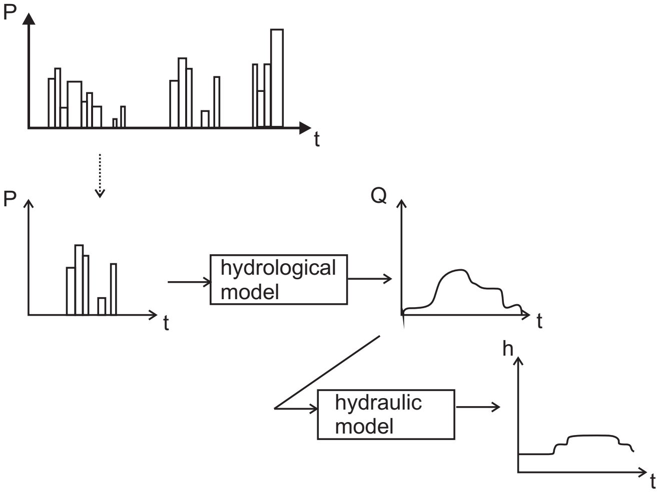
A need for ensemble generation of design storms



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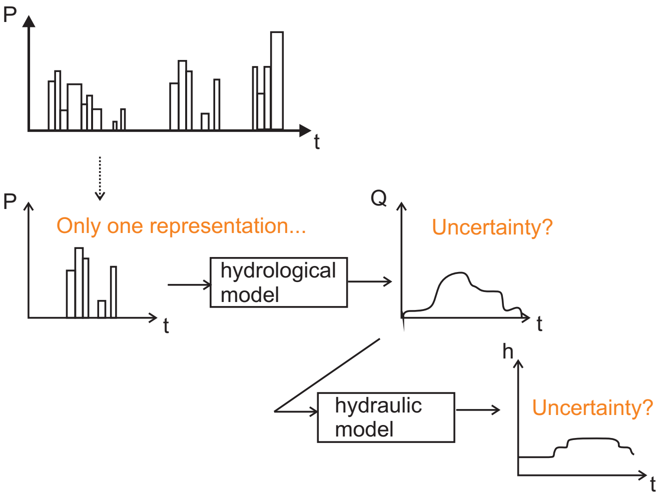
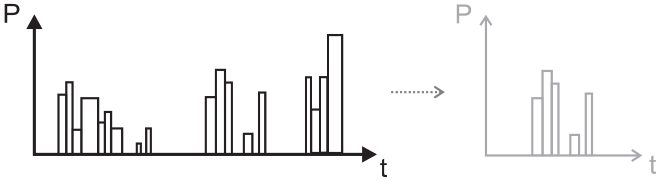


A need for ensemble generation of design storms

Aim

A methodology to generate an ensemble of storms, representative for one historical storm in terms of their extremity and internal storm structure.

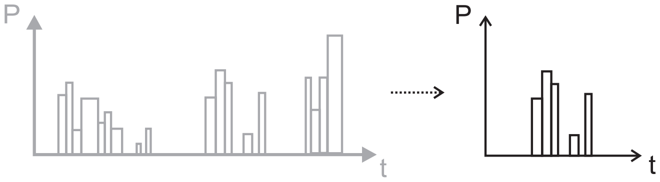
Storm selection



Time series: 105 years of Uccle rainfall (Brussels) with a 10-minute resolution

24 hour dry period selection criterion (Poisson storm arrivals)

Storm selection



Winter, Spring, Summer or Autumn storm

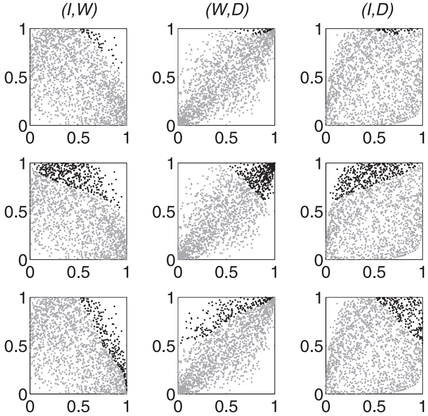
1st-, 2nd-, 3rd- or 4th- quartile storm

Most useful storm variables for a bivariate description of storms?

Storm selection

Normalized rank-scatter plots for pairwise dependence analysis

mean storm intensity I [mm/h], storm duration W [h] and total storm depth D [mm]



Storm selection

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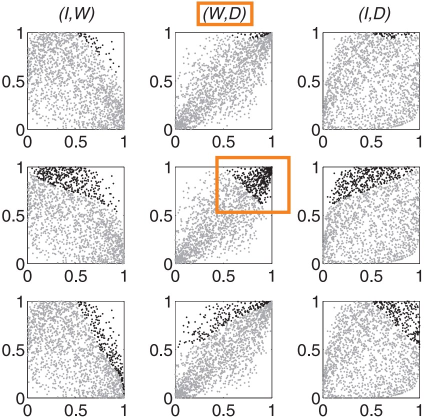
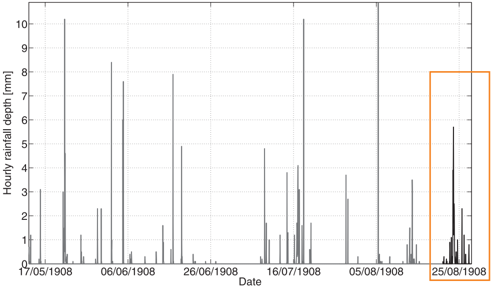


Storm selection

A storm is described by W and D, which are associated

A bivariate joint distribution function of W and D should incorporate this association

One observed storm of interest



21st of August 1908; W = 153 h ; D = 43.3 mm; 2nd-quartile

What is its return period and internal storm structure?

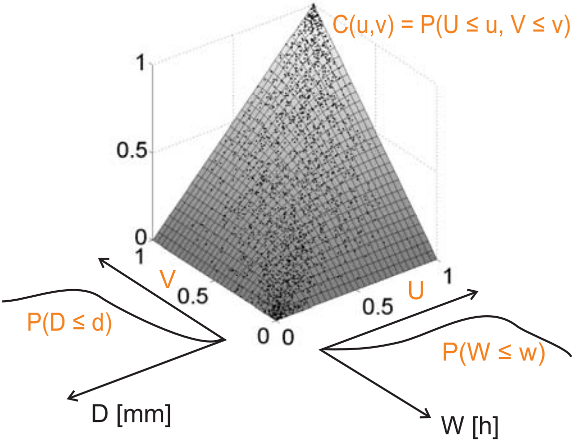
Simulate 10 000 'statistically equal' storms

Return period of observed storm

To calculate a probability of occurrence of a storm, the bivariate cumulative distribution function (CDF) of W and D is constructed using copulas

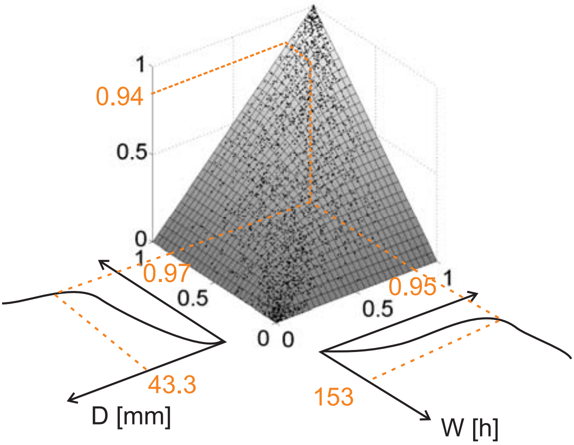
Return period of observed storm

A copula links the two marginal cumulative distribution functions (CDFs) of W and D into a bivariate CDF



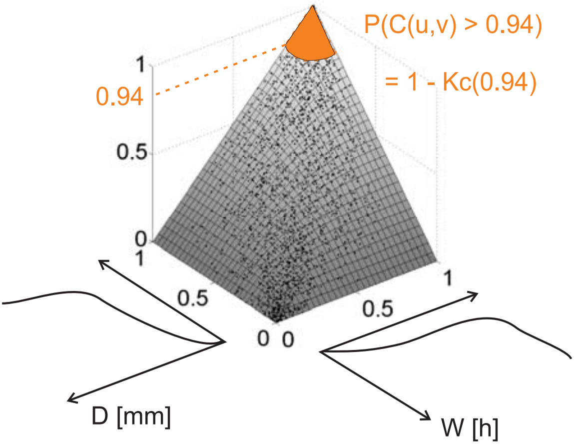
Return period of observed storm

The observed storm corresponds with a certain probability level of the copula



Return period of observed storm

The probability that a storm occurs at a higher copula-level t is used for the calculation of the return period



Return period of observed storm

The (secondary) return period:

$$T = \frac{\text{mean storm interarrival time}}{\mathbb{P}(\text{summer storm}) \cdot (1 - K_{\mathbf{C}}(t))}$$

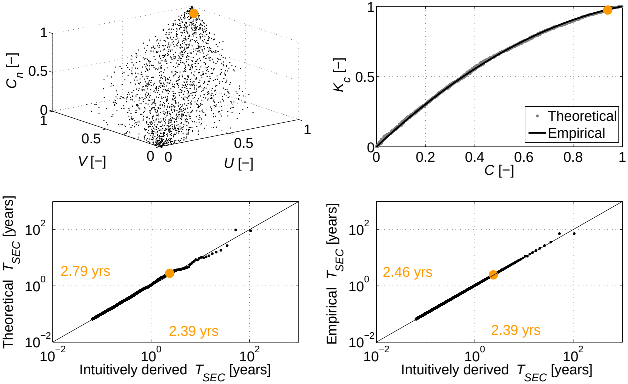
The function $K_{\mathbf{C}}$ can be calculated theoretically or empirically

The return period can also be intuitively derived

Are there differences?

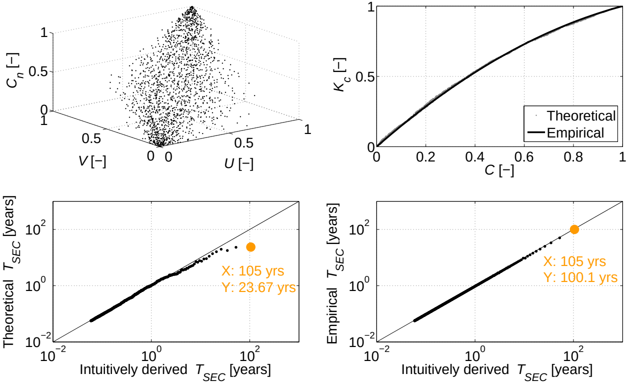
Return period of observed storm

Good correspondence between intuitively derived return period and the theoretical or empirical one

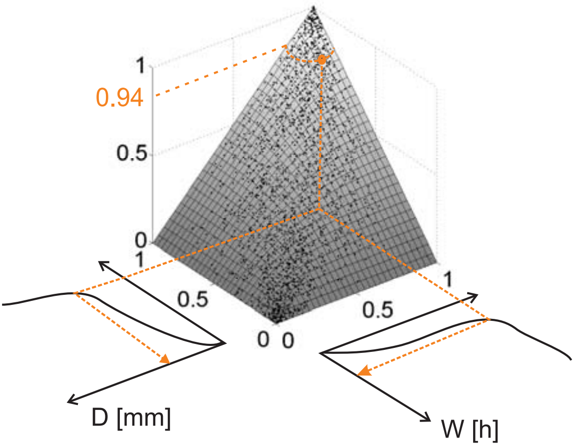


Return period of observed storm

But...a worse fit of the copula for winter storms gives large differences



Simulation of equally extreme storms



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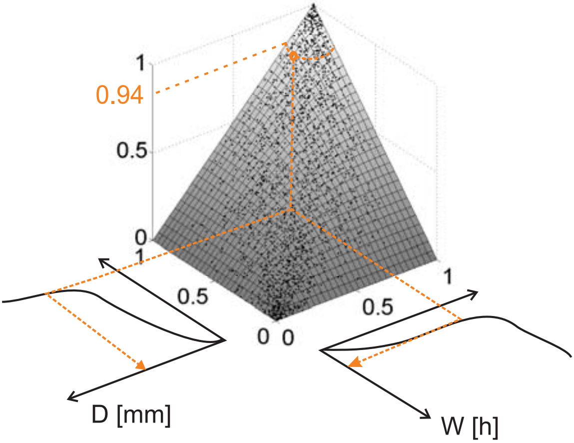

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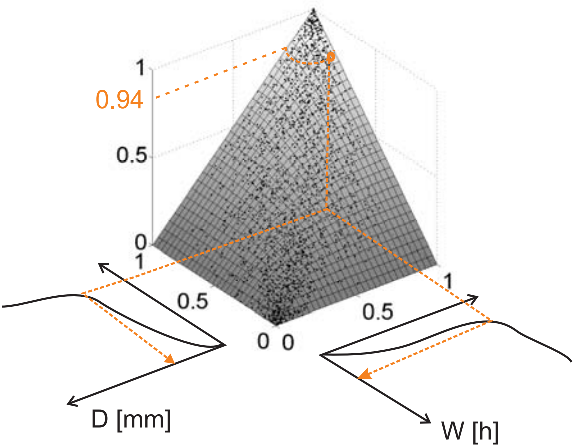
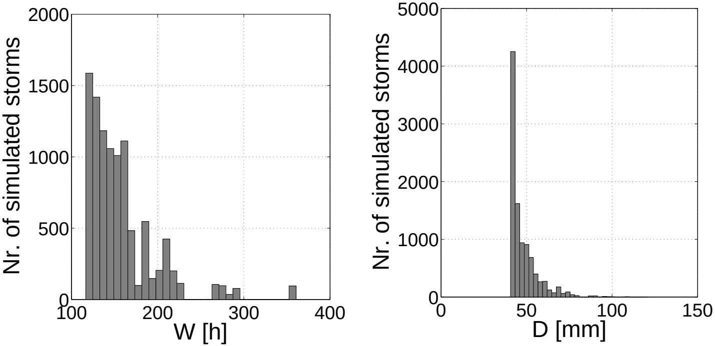


Simulation of equally extreme storms

10 000 simulated equally extreme storms - based on observed summer storm with $W = 153$ h and $D = 43.3$ mm



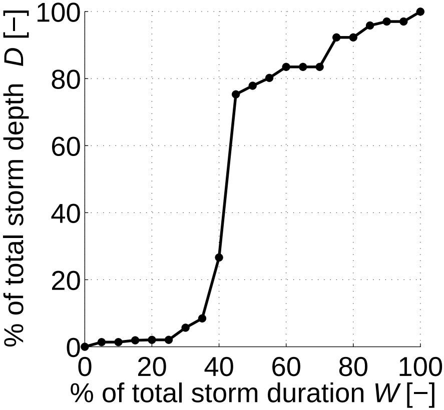
Internal storm structure of observed storm

Concept of mass (Huff) curves is used to describe internal storm structure:

Percentage of cumulative storm depth at 5% time intervals of the total storm duration

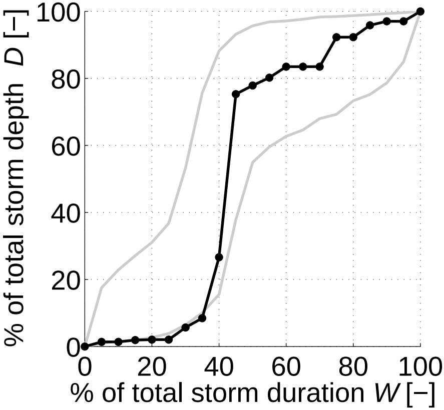
Internal storm structure of observed storm

The internal storm structure of the observed storm



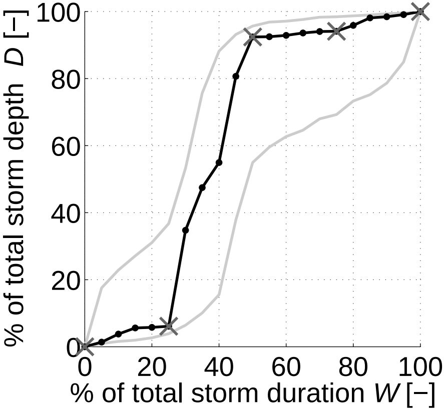
Internal storm structure of observed storm

10% and 90% percentile curves for 2nd-quartile summer storms



Internal storm structure of observed storm

Random simulations of internal storm structure



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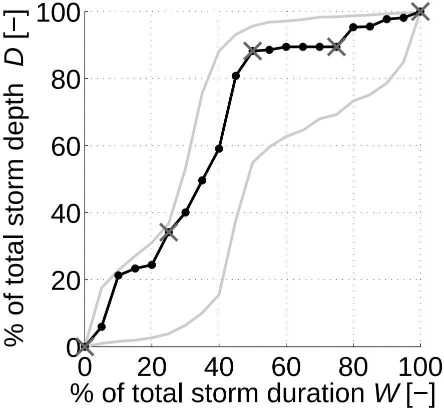

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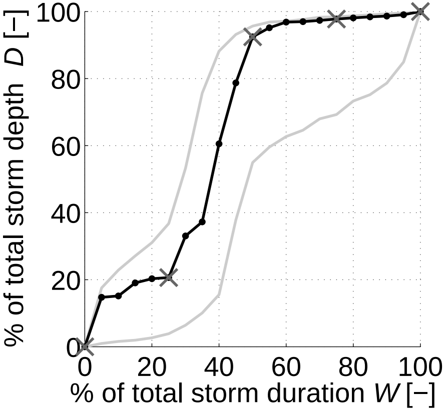

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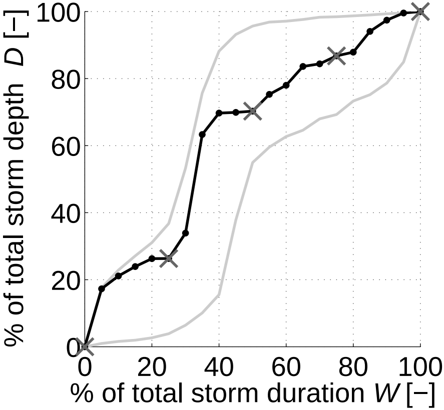

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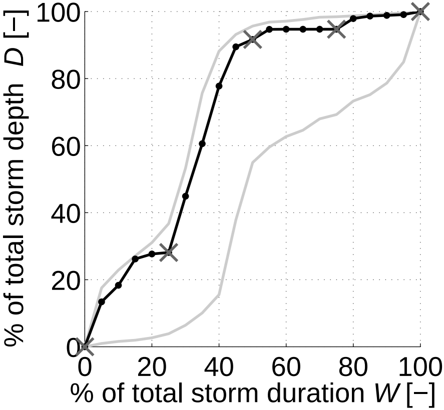

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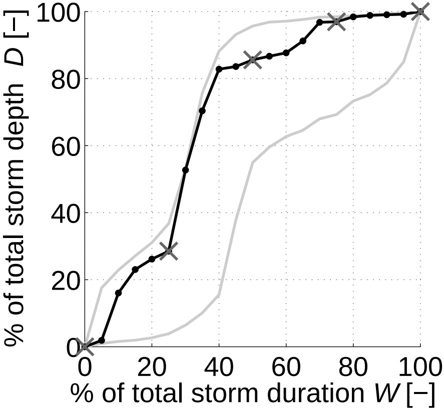

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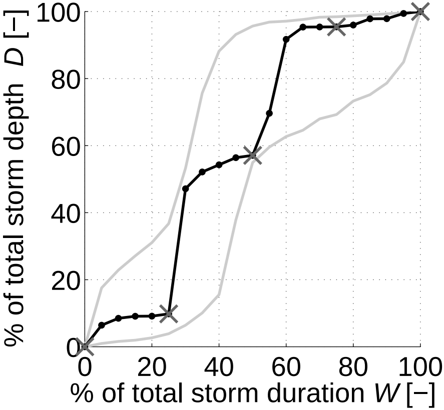
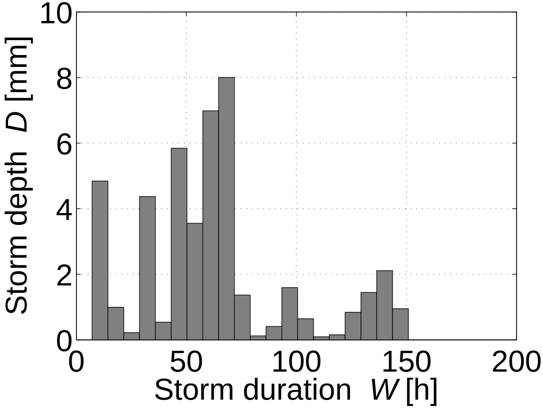


An ensemble of predefined design storms

Superposition of 10 000 randomly generated internal storm structures on W and D of the 10 000 simulated storms.



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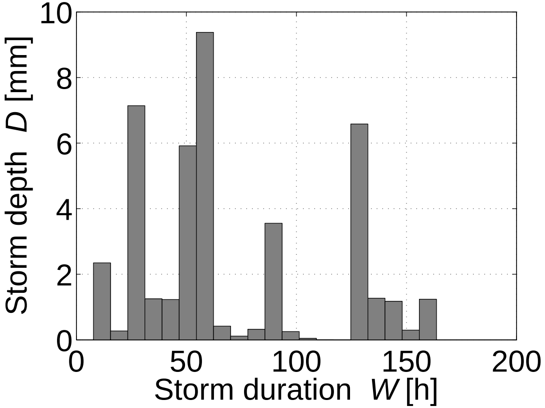

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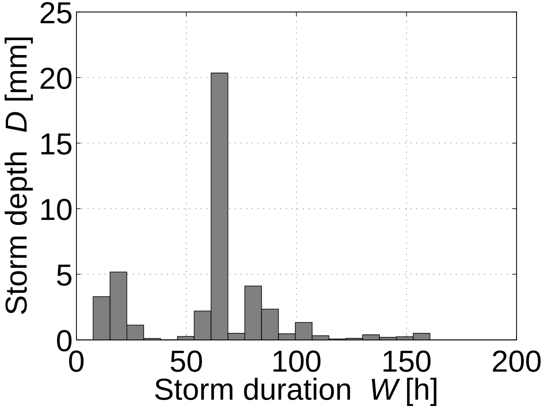

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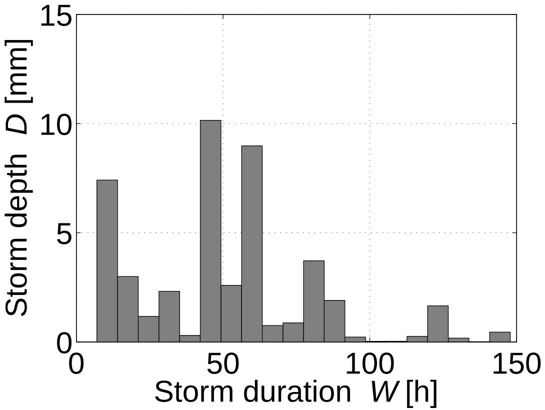

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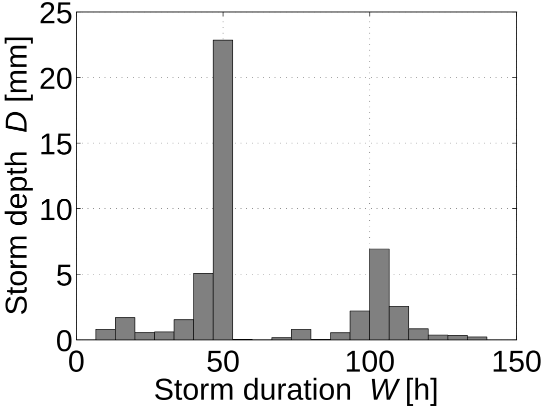
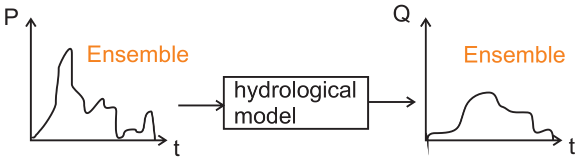


Hydrological simulation with ensemble of design storms

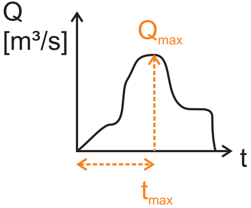


The Probability Distributed Moisture (PDM) model of the Demer catchment (Belgium) is forced with the simulated ensemble of design storms

The same antecedent soil moisture conditions are maintained by using the same historical storms before the design storm.

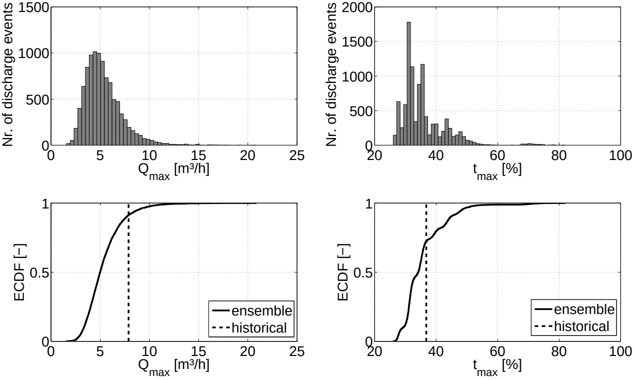
Hydrological simulation with ensemble of design storms

Several characteristics of the simulated discharge can be evaluated



Hydrological simulation with ensemble of design storms

The simulation with the ensemble allows for an evaluation of the uncertainty on the historical discharge.



Conclusions and future perspectives

The potential of a stochastic design storm generator, which combines the concepts of a copula-based return period and mass curves, is demonstrated in a practical setting

The proposed methodology is easy and fast

Other areas of application, including design of hydraulic structures, should be studied

Advances in theoretical copula research (goodness-of-fit, asymmetry, multidimensionality, . . .) could improve the stochastic design storm generator