Development of a method of rain gauge network optimization based on Intensity-Duration-Frequency results

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Abstract

In this work, a method using intensity-duration-frequency curves (IDF) is developed in a context of robust optimisation in order to identify the best locations to implement new rain gauges with respect to rainfall variability characteristics. The advantage of robust optimisation is to derive design solutions that yield networks which work 'adequately', when considering various rainfall events. Thus, the robust optimisation approach overtakes the problem of selecting representative rainfall events to build the optimisation process. The presentation develops an original methodology based on Montana IDF model parameters. The latter are assumed as geostatistical variables and their spatial interdependence is taken into account through the adoption of cross-variograms in the kriging process. To assess the optimal location of new monitoring stations with, as point of departure an existing rain gauge network, the objective function is based on the mean spatial kriging variance and the variogram model using the so-called variance-reduction method. Various return periods as well as two time horizons (short term and long-term) are considered to built a robust objective function which is further minimized using simulated annealing algorithm. Application concerns the case study of north Tunisia augmentation rain gauge network. Results of IDF curves estimated using a network of 14 stations are analyzed. Robust optimization optimal networks are compared to those obtained assuming the following mono objective criteria: minimization of the mean spatial kriging variance of rainfall intensity and erosivity factor patterns using data from a single extreme rainfall event occurred in March 1973.

Objectives

- ➢ develop a methodology of rainfall network optimization based on the well-known Montana IDF model.
- ➢ derive design solutions that yield networks which work 'adequately', when considering various rainfall events
- ➢compare results to those of classical mono objective criteria based on the analysis of 1h-rainfall intensity observations and erosivity factor observations during a given extreme rainfall event.

Methodology

❖ A number of candidate stations is first assumed. The analysis is run for a giving size of the optimal network. The problem is to find the best locations. On the other hand, the point of departure of the study is an existing network of 13 stations which will be extended.

❖ **The IDF curves Montana model are adopted to conduct the analysis:** $I(t,T)=a(T)*t^{-b(T)}$

- the dependence structure between the parameters a(T) and b(T) by is described using their cross variogram $\gamma_{a(T)b(T)}$:

$$\gamma_{a(T)b(T)}(h)=\frac{1}{2*N(h)}\sum_{i=1}^{N(h)}\left[a(x_i+h)-a(x_i)\right]\left[b(x_i+h)-b(x_i)\right]$$

and codispersion coefficient graph $r_{a(T)b(T)}(h)$:

$$r_{a(T)b(T)}(h)=\gamma_{a(T)b(T)}(h)\Big/\sqrt{\gamma_{a(T)a(T)}(h)*\gamma_{b(T)b(T)}(h)}$$

- The variable of interest is set to be a(T) by considering sequentially six return periods (*T*= 2, 5, 10, 20, 50, 100 years) thus covering a broad panoply of risk situations.

❖ **Kriging is adopted as method to design the optimal network:**

- Kriging with external drift is used to derive the parameter a(T) throughout the study area; parameter b(T) obtained by ordinary kriging is assumed as external drift

❖**Two time horizons:** in short term (N= 5 years) and in long-term (N= 30 years)) are considered to built a robust objective function which is further minimized using simulated annealing algorithm

❖ **The objective function** (*OF*) is based on the mean spatial kriging variance with a grid mesh of 4 km size (covering an area of 21000 km²) using the so-called variance-reduction method:

$$OF=Prob_{(2years)}*(OF_{(2years)}-OF_{ref(2years)})^2+Prob_{(5years)}*(OF_{(5years)}-OF_{ref(5years)})^2+Prob_{(50years)}*(OF_{(50years)}-OF_{ref(50years)})^2$$

Where $Prob_{(Tyears)}$ is exceedance probability of the event of return period T (in years) with $Prob_{(2years)}+Prob_{(5years)}+Prob(_{50years})=1$

- Standardization of the mean spatial kriging variance is obtained by using the interquartile difference of the field of the variances of error of *a(T)* :

$$\mathrm{FO}(\mathrm{T\ ans})=\left(\sum_{i=1}^{n}\left(\sigma_{i(a(Tans))}\right)^2\Big/n\right)\Big/\left(\sigma^2_{75\%(a(Tans))}-\sigma^2_{25\%(a(Tans))}\right)$$

$\sigma^2_{75\%(a(Tans))}$ is the 75% percentile of the pattern of the variance of kriging errors of *a*(*T* ans)

$\sigma^2_{25\%(a(Tans))}$ is the 25% percentile of the pattern of the variance of kriging errors of *a*(*T* ans)

$OF_{ref}(T\ ans)$ is the value of the objective function obtained for every return period independently of the other return periods. It is taken as reference.

❖ **Various Scenarios** are simulated where the size of the initial network is increased by 25 (Scenario1), 50 (Scenario 2) and 100 % (Scenario 3), considering 40 candidate stations. Another application is shown for increasing the network to meet WMO requirement, considering 60 candidate stations (Scenario 4). The two time horizons are simulated for each scenario.

Case study and Data

- Data cover the North of Tunisia area (Mediterranean coast belonging to Medjerda basin)
- **Figure 1** reports the study area together with the pluviograph networks involved in 1973 extreme event and candidate stations.
- Intensity-duration-frequency curves (IDF) are reported for 14 stations of the study area according to the study of ST2i (2007).
- Montana IDF model parameters estimated by ST2i (2007) are presented in **Table 1**

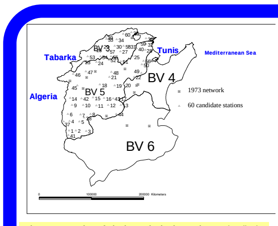


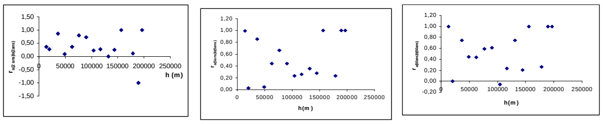
Figure 1: Location of pluviographs in the study area (Medjerda basin *BV5*; Northern coast basins *BV3*; Meliane Basin *BV4*; Central Tunisia basins *BV6*)

Identification	2 ans		5 ans		10 ans		20 ans		50 ans		100 ans	
	a(T)	b(T)	a(T)	b(T)	a(T)	b(T)	a(T)	b(T)	a(T)	b(T)	a(T)	b(T)
Ghardimaou	231.6	0.685	289.9	0.69	324.4	0.688	356.9	0.686	401.3	0.685	435.7	0.685
Zouarine Gare	240	0.66	303	0.64	352	0.63	404	0.63	479	0.62	542.0	0.62
Ain Taga	437.1	0.538	541.8	0.53	614	0.524	680	0.517	759.9	0.507	812.4	0.498
Ain Beya Ferrana	368.5	0.682	528.6	0.694	629.6	0.683	715	0.665	804	0.635	856.4	0.61
Haidra Poste Douanes	321.4	0.745	406	0.741	456.9	0.734	503.4	0.726	556.9	0.713	594.4	0.704
Izid Barrage	157.7	0.652	307.7	0.73	465.9	0.776	651.5	0.81	920.4	0.84	1142.3	0.857
Joumine Antra	179	0.549	220	0.511	249.5	0.49	277.7	0.473	313.8	0.455	341.8	0.445
Mellègue K13	410.4	0.722	553	0.68	668.7	0.654	793.3	0.635	965.1	0.616	1103.9	0.606
Oued Tine cassis	144.5	0.626	206.3	0.637	241.9	0.624	286.8	0.618	345.9	0.605	392.4	0.594
Sarrat Pont Route	431.5	0.758	688.3	0.789	848.3	0.796	992.5	0.798	1171.5	0.797	1317.5	0.798
Sejnène	306.8	0.669	349.2	0.663	385.4	0.66	429.2	0.661	499.2	0.666	559.9	0.671
Siliana Laouej	256.2	0.635	342.6	0.675	406	0.692	461.1	0.698	512.4	0.689	530.8	0.67
Slouguia	275.1	0.695	343.8	0.69	387.7	0.682	427.6	0.673	479.7	0.663	539.8	0.657
Sraya Ecole	162.1	0.592	200.1	0.592	237.2	0.596	284.7	0.604	367.1	0.619	448.1	0.633

Table 1: Parameters *a(T)* and *b(T)* for the 14 studied stations for IDF (ST2i, 2007)

Results

- **(a)** Interdependence of the parameters a(T) and b(T) as reflected by erratic fluctuation of $r_{a(T)b(T)}(h)$ (**Figure 2**) ; $r_{a(T)b(T)}(h)$ are assumed constant for all T values.



For T= 2 years

For T= 5 years

For T= 50 years

Figure 2: Fluctuation of $r_{a(T)b(T)}(h)$ supports interdependence assumption

- **(b)** Structural analysis of the parameters a(T) and b(T) results in three groups where the variograms are similar : **group 1 {*T*= 2 years}** ; **group 2 {*T*= 5, 10, 20 years}**; **group 3 {*T*= 50, 100 years};** Representatives of each group are: T=2 years, T=5 years and T=50 years; Ranges of a(T) and b(T) variability extent from 30 km to 50 km **(Table 2)**

Return period (years)	Sill a (Sill b)	Range a km (Range b km)
2	0.004 (15000)	30 (40)
5	0.008 (32000)	40 (50)
10	0.010 (45000)	40 (45)
20	0.014 (50000)	40 (50)
50	0.016 (110000)	40 (50)
100	0.018 (110000)	40 (50)

Table 2: Adjusted sills and ranges for a(T) and b(T) assuming spherical variogram models

(c) The comparison of the resulting robust networks obtained successively for the two time horizons (short term and long-term) shows that they are similar for scenario 1. For scenario 2, the two robust networks differ only by one station over the two time horizons (short term and long term). For scenario 3, the two robust networks obtained successively for the short term and long term horizons differ only by 2 stations out of 13. For scenario 4, they differ by 7 stations out of 21 (**Figure 3**).

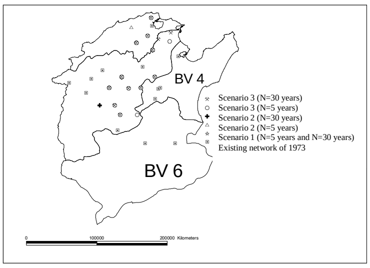


Figure 3: Configuration of optimal networks

(d) The comparison of the robust networks to the mono objective optimal networks obtained successively for the rainfall intensity and the erosivity factor mapping for March 1973 shows that the resulting networks are very different for the two horizons and for all scenarios. Results reflect the decreasing of the objective function when the size of the final network (**Figures 4 and 5**) is increased and that the gain in augmenting the network size is less perceivable using the robust estimation than using the mono objective criteria. The latter give rise to optimal objective function values that are lower or equal to that obtained in the case of the robust optimisation (**Figures 4 and 5**).

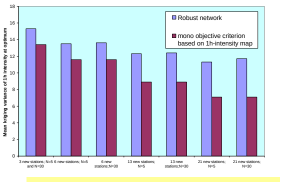


Figure 4: Mean spatial kriging variances obtained by the robust network and by the mono objective criterion for Rainfall intensity mapping

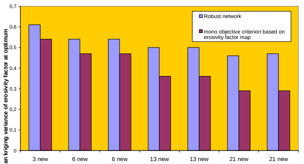


Figure 5: Mean spatial kriging variances obtained by the robust network and by the mono objective criterion for Erosivity factor mapping

Conclusions

The robust optimization approach considering IDF curves parameters and kriging was applied to locate the best sites for implementing new raingages North Tunisia. The method is based on the choice of a time horizon and on objective functions involving IDF-return periods and parameters. Accordingly, robust optimization results are compared to two mono objective criteria (rainfall intensity mapping and erosivity factor mapping) using one single extreme rainfall event analysis. The comparison of results highlights that the percentage of gain in the robust optimisation is lower or equal to that obtained in the case of the mono objective criterion. Nevertheless, the most advantage of the robust optimisation lies in the fact that it allows to overcome the problem of elaborating rainfall event database in case of adopting the classical approach of variance-reduction method based on single rainfall patterns.

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