Uncertainty Estimation of hydrological models by using Gaussian-Process-Regression

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Kos, 19. October 2013

Introduction

Hydrological models are subject to several uncertainties

- **Sources of uncertainty are for example:**

Data uncertainty
- measurement errors
- regionalization of data

Structural uncertainty
- the model is always only a simplified representation of real world processes
- not all processes can be taken into account

Parameter uncertainty
- different sets of model parameters are equally likely

- Uncertainty of the model determines whether a model is suitable for a specific application

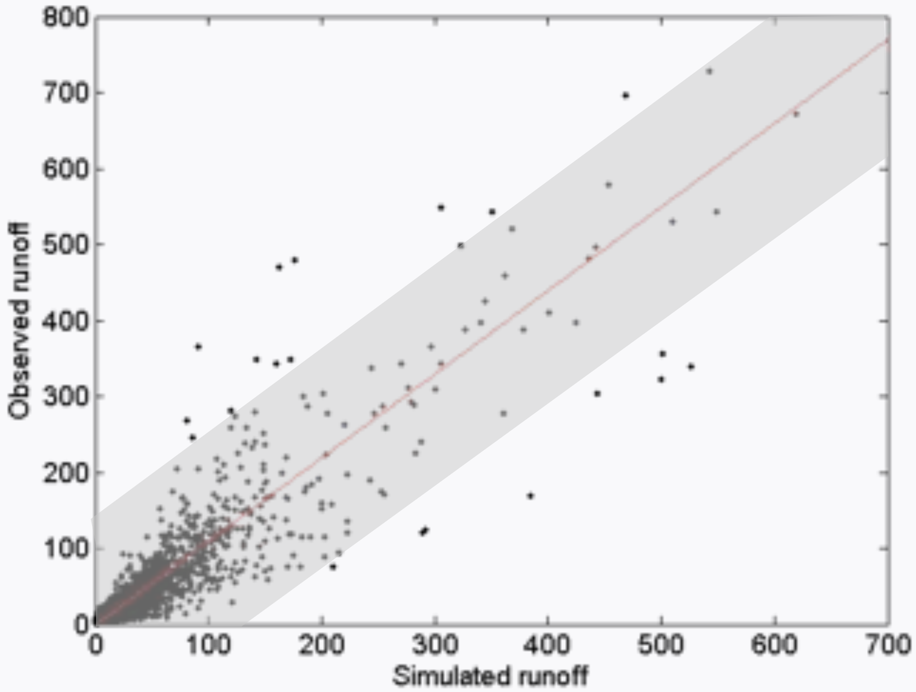
Problem definition

- **Problem:** Estimation of confidence intervals for the simulation of runoff where long measured runoff time-series are available
- **Statistical properties of the model error:**
  - heteroscedastic
  - non-gaussian
  - non-stationary
  - auto-correlated (i.e. not independent)

Properties of the model error

- **Approach:** Predict the observed runoff from the simulated runoff by using methods for function regression

=> Estimation of the prediction error



- Examples of possible methods
  - Linear regression
  - Meta-Gaussian Model (Montanari & Brath, 2004)
  - **Gaussian-Process-Regression (GPR)**

Gaussian-Process-Regression

- Training data are $\boldsymbol{D} = \{\boldsymbol{x}^{(i)}, \boldsymbol{o}^{(i)} \mid \boldsymbol{i} = \mathbf{1}, \dots, \boldsymbol{n}\}$
- Each input is the simulated runoff ***x***.
- Each observation is a real-valued scalar, which follow a noisy function

o = f (x) + noise.

- Collect inputs in 1 × n matrix, ***X***, and targets in vector, y:

D = {X, y}

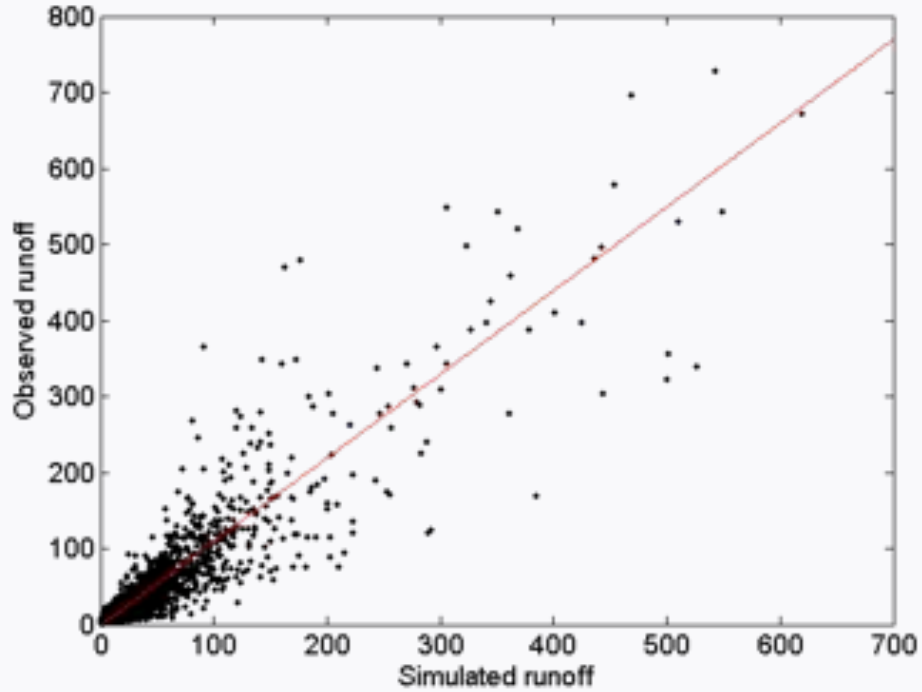
- Wish to infer $\boldsymbol{y}^*$ for unseen input $\boldsymbol{X}^*$, using ***P(y*|X*,D)***
- **Gaussian Process Regression (GPR)** gives $P(\boldsymbol{y}^*|\boldsymbol{X}^*,\boldsymbol{D}) = N(\mu, \sigma^2)$

$$\mu = (K(\boldsymbol{X}^*, \boldsymbol{X})K(\boldsymbol{X}, \boldsymbol{X}) + s^2 I)^{-1}\boldsymbol{y}$$

$$\sigma^2 = K(\boldsymbol{X}^*, \boldsymbol{X}^*) - K(\boldsymbol{X}^*, \boldsymbol{X})[K(\boldsymbol{X}, \boldsymbol{X})s^2 I]^{-1}K(\boldsymbol{X}, \boldsymbol{X}^*)$$

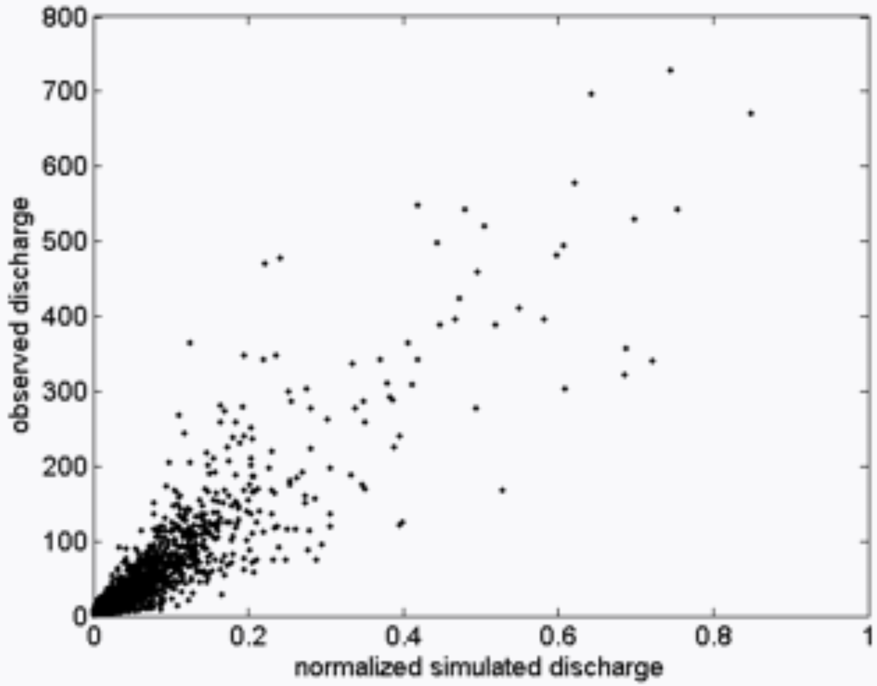
(Rasmussen & Williams, 2006)

Preprocessing steps



Preprocessing steps

Step 1: Linear rescale of simulated discharge

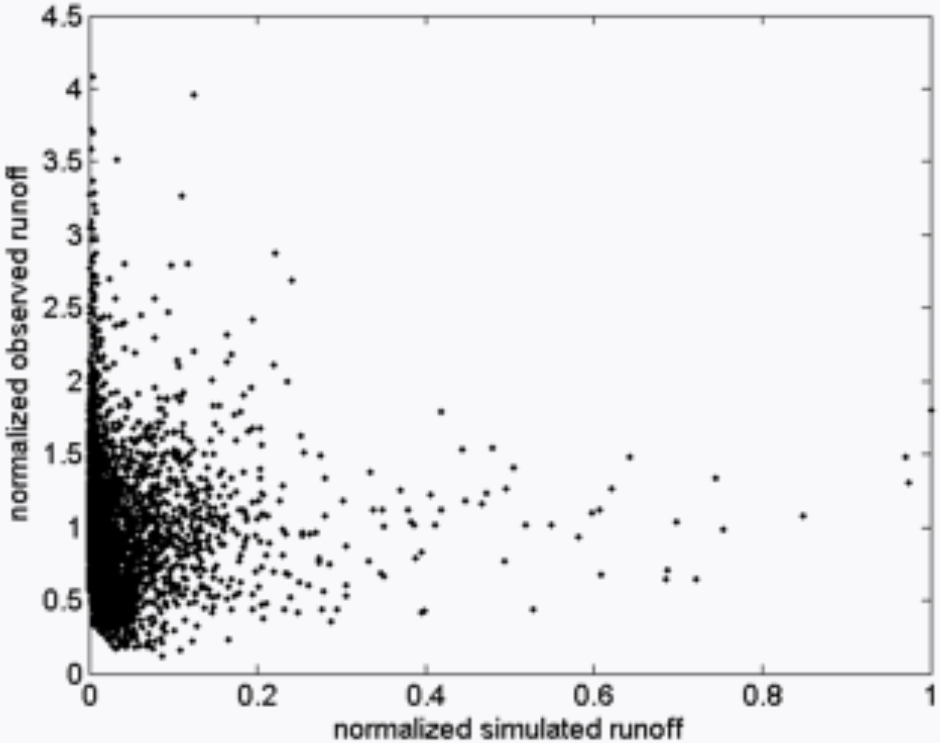


Preprocessing steps

Step 1: Linear rescale of simulated discharge

Step 2: Normalization of the observation

$$o'^{(i)} = \frac{o^{(i)}}{x^{(i)}+\beta}$$



Preprocessing steps

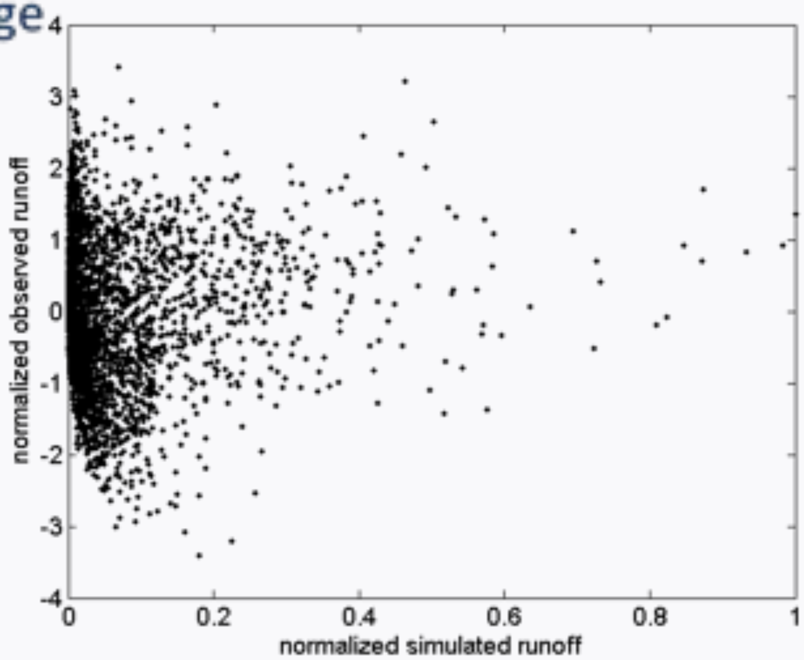
Step 1: Linear rescale of simulated discharge

Step 2: Normalization of the observation

$$o'^{(i)} = \frac{o^{(i)}}{x^{(i)} + \beta}$$

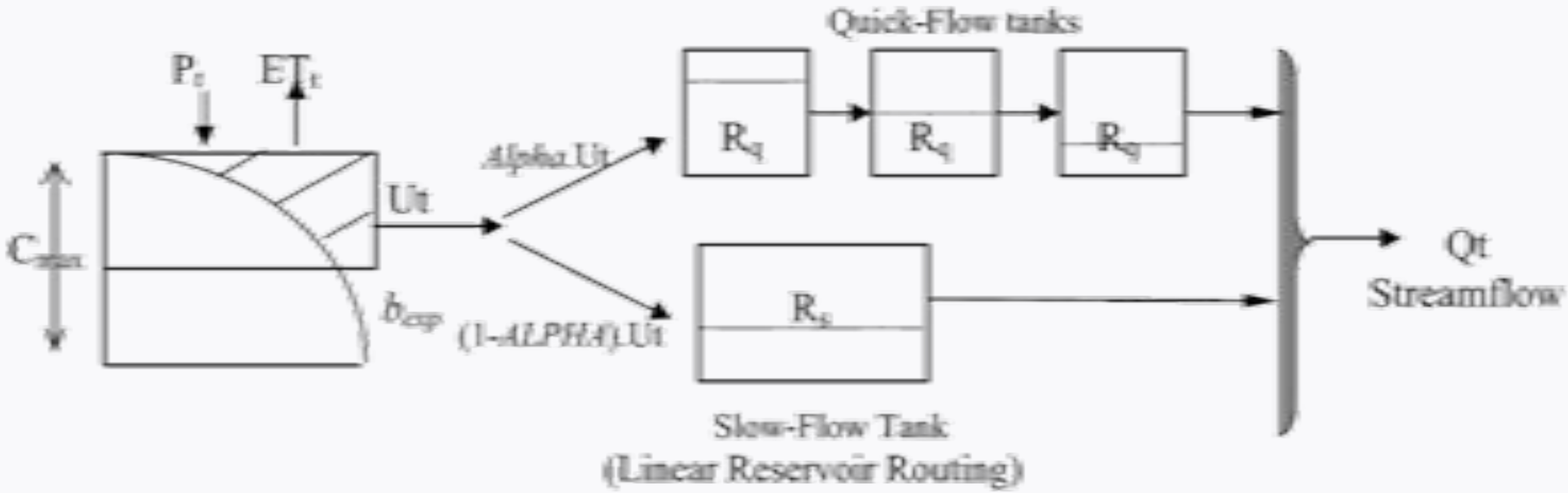
Step 3: Transformation by a Normal Quantile Transformation (NQT)

$$o''^{(i)} = NQT\left(o'^{(i)}\right)$$



Example

Hydrological model HYMOD



(Boyle, 2001)

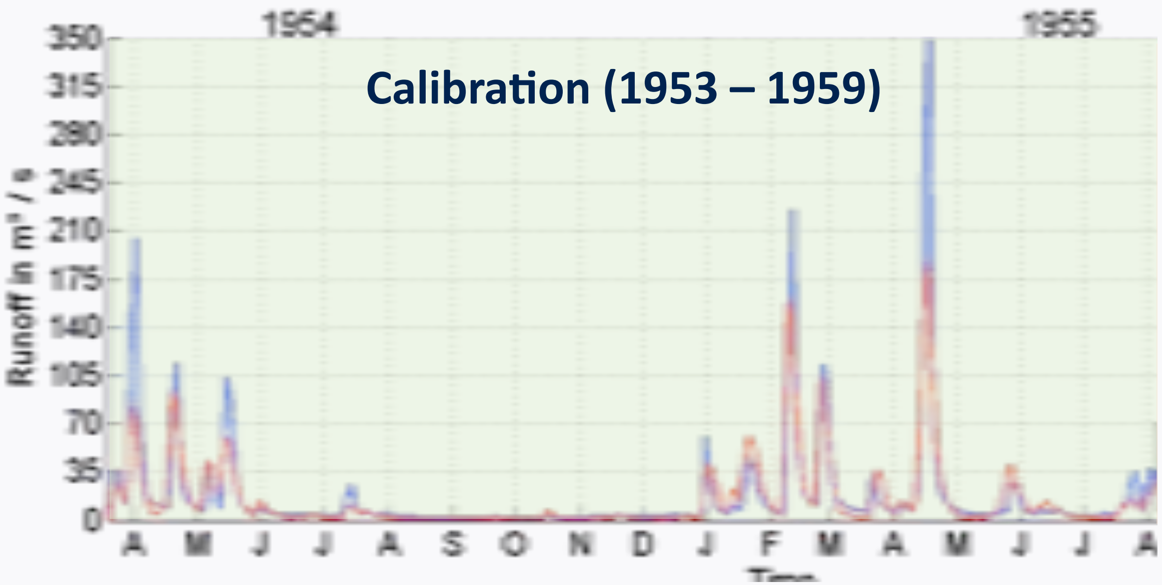
Example

- **Leaf River catchment**

- Used in studies by Vrugt et al. (2003-2013), Ajami et al. (2007), Misirili et al. (2003)

Example

Hydrological model HYMOD



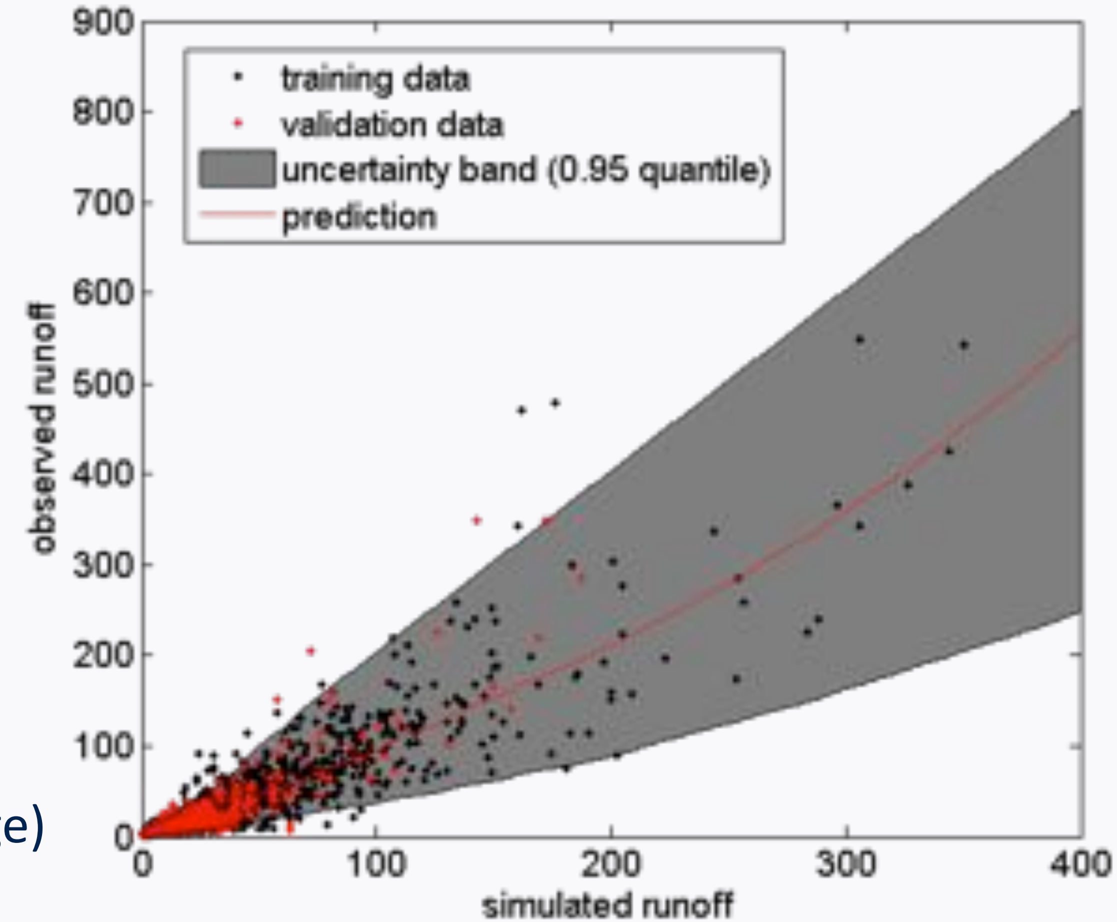
Calibration (1953 – 1959)

Validation (1960 – 1962)

Preliminary result

Gaussian Process Regression

- prediction increases linearly for simulated runoff up to 200m³/s and more than linear after that
- 91% of all observations are within the uncertainty range
- average error is 68 m³/s (124.23% of average discharge)



Enhancements

- Gaussian Process Regression is a multivariate method

=> use several model states for prediction of observed runoff

- Possible candidates:
  - simulated discharge (Q)
  - rainfall input (R)
  - change of discharge in last 7 days (Q7)
  - total precipitation in last 7 days (R7)

model states	**observations within uncertainty band**	**average range of uncertainty band**
Q	91%	68 m³/s
Q,R	92%	64 m³/s
Q,Q7	90%	66 m³/s
Q, R7	91%	65 m³/s
Q,R, Q7	91%	65 m³/s
Q, R, R7	92%	64 m³/s
Q, Q7, R7	91%	65 m³/s
Q, R, Q7,R7	91%	66 m³/s

Final result

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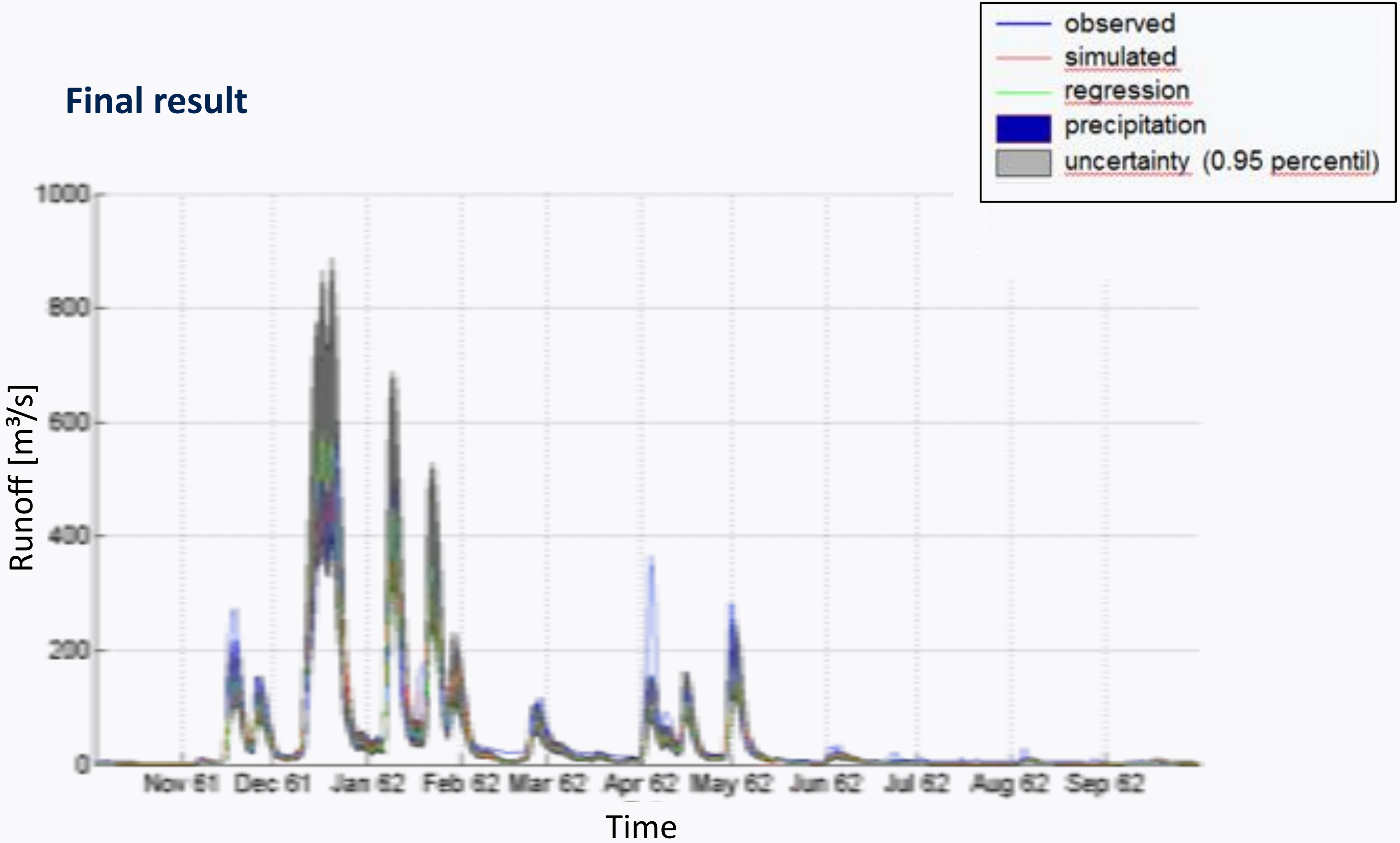
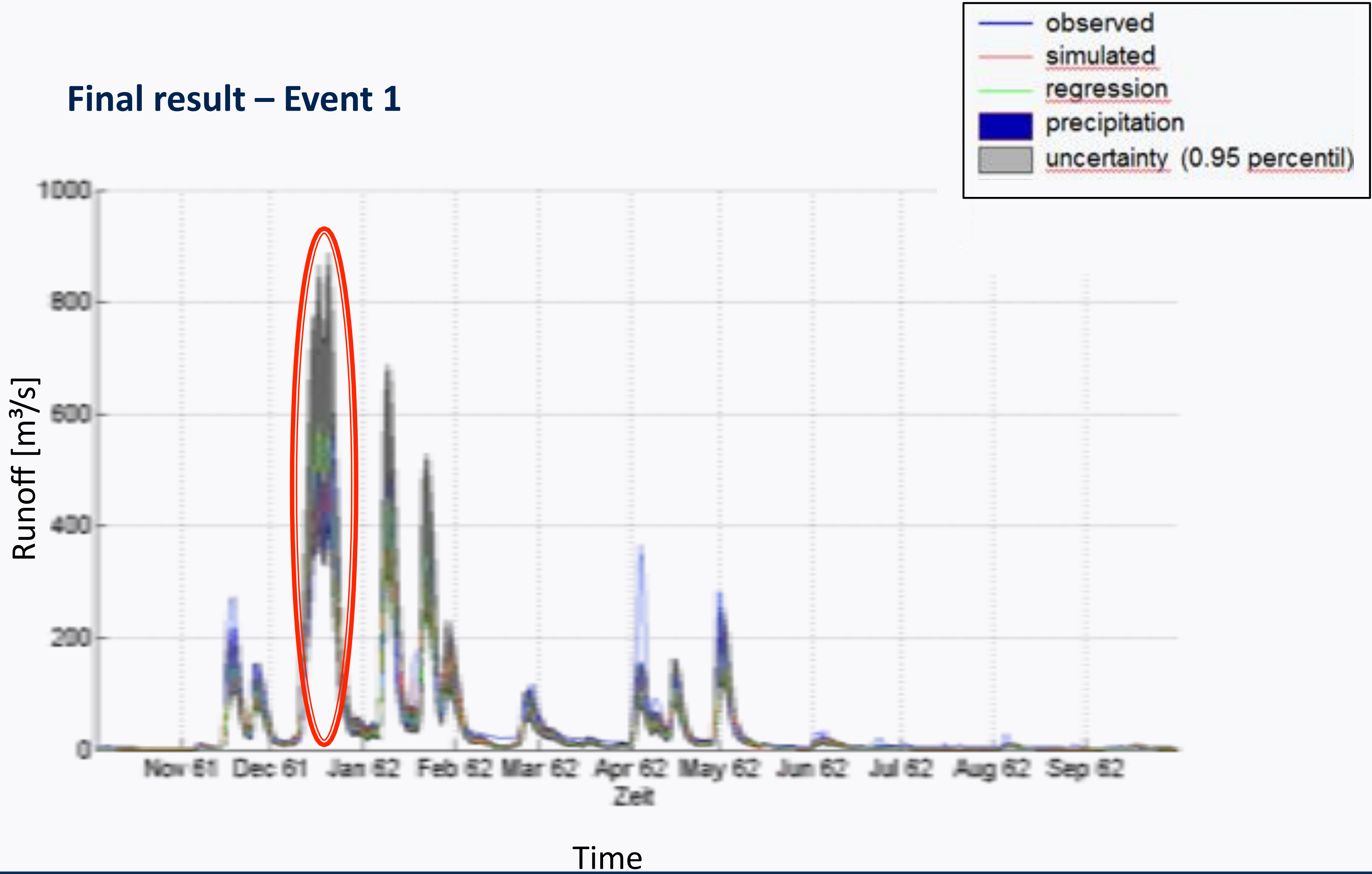


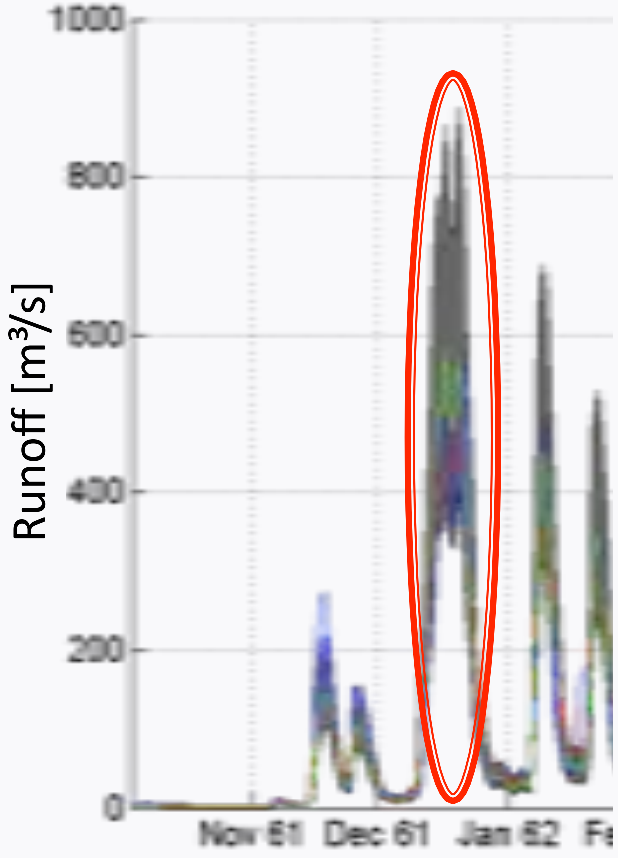
Final result

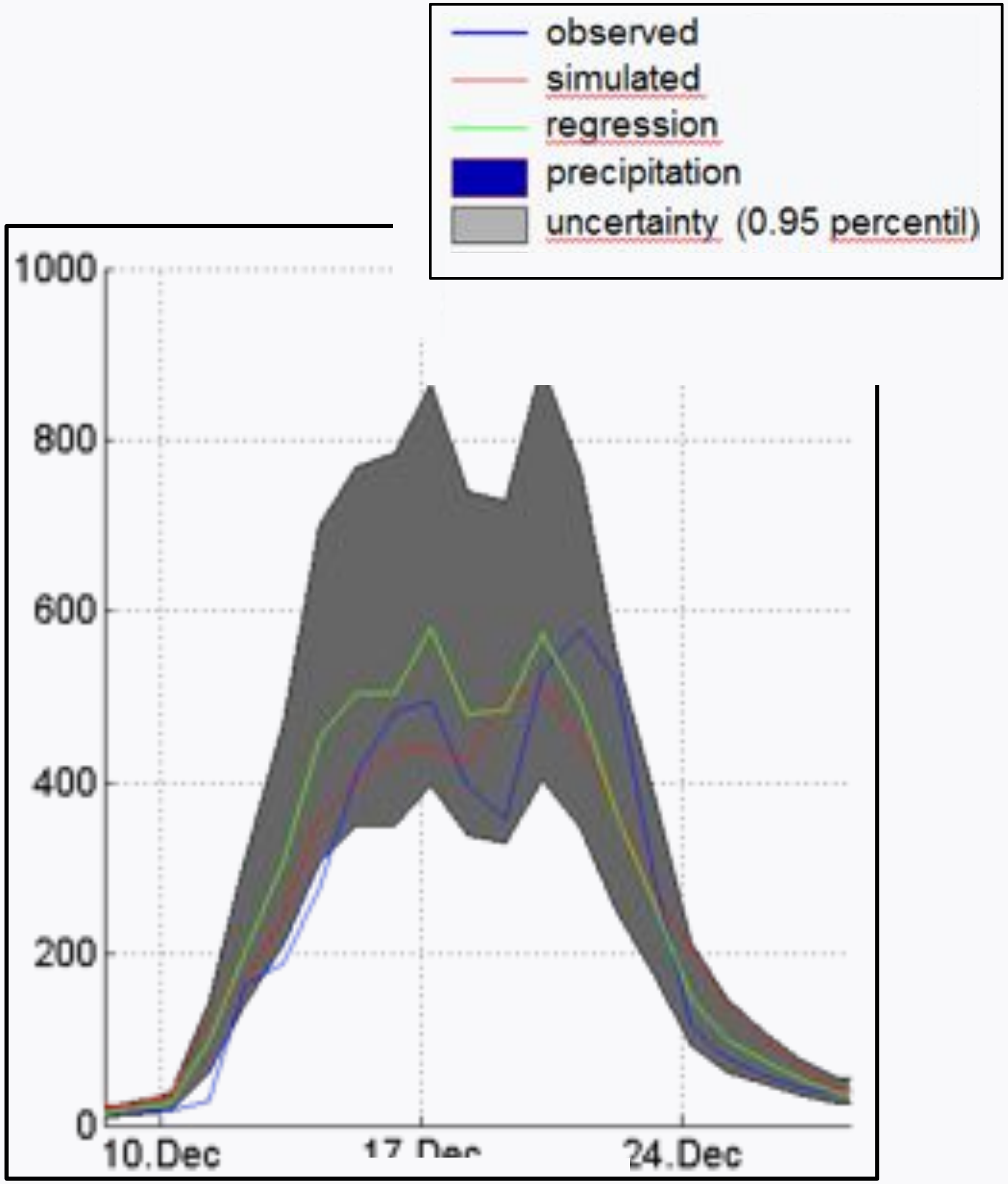
Final result – Event 1



Final result

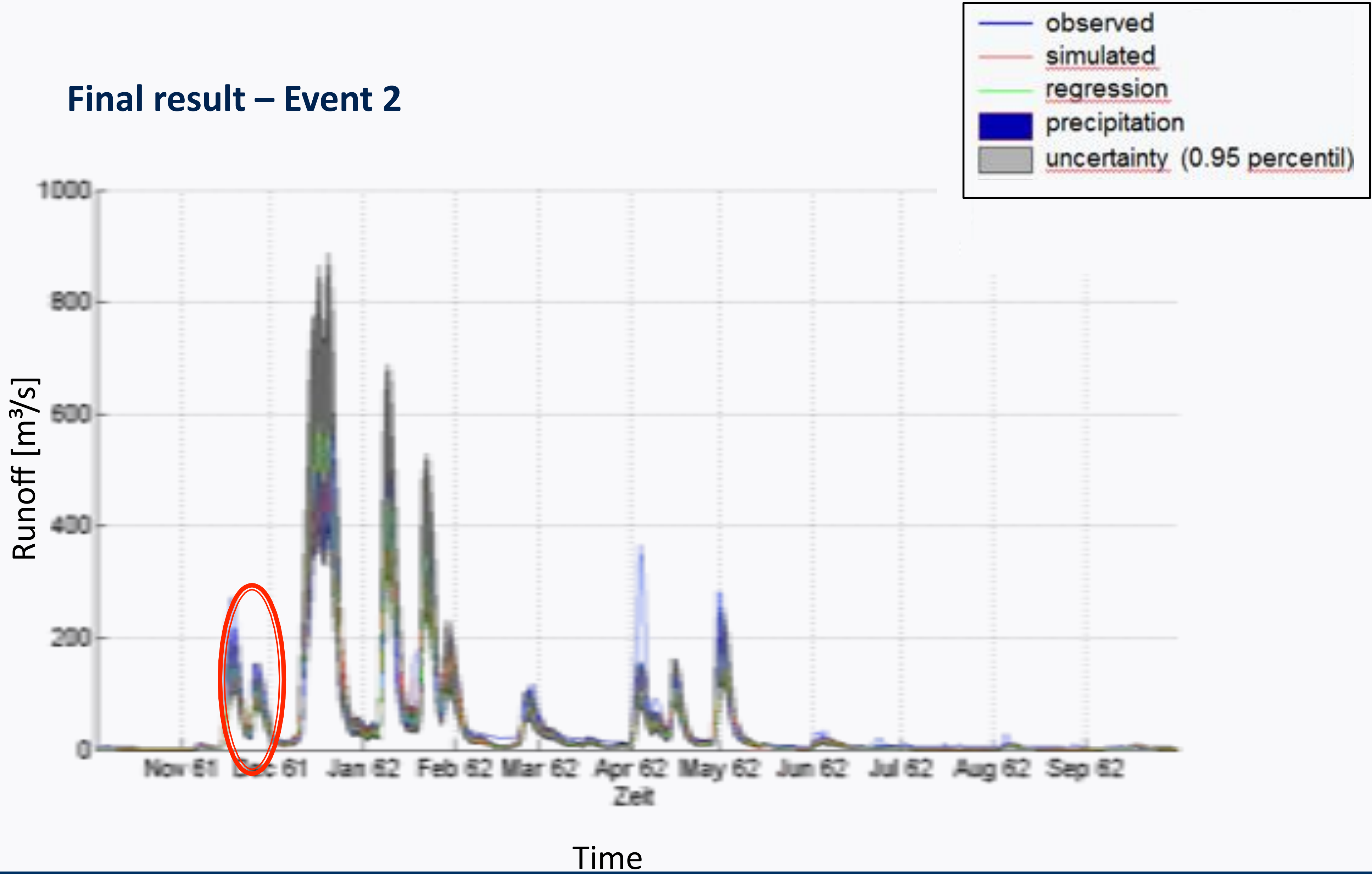
Final result – Event 1





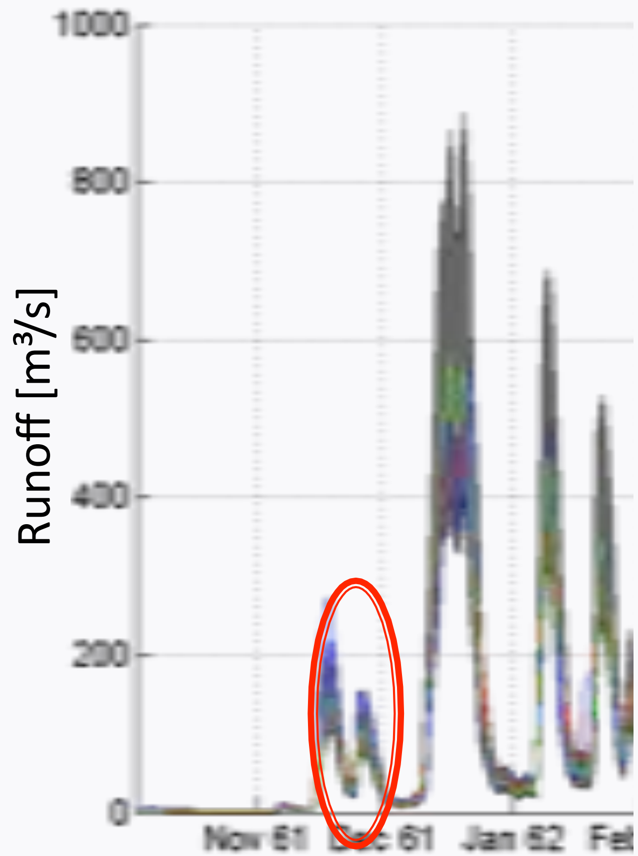
Final result

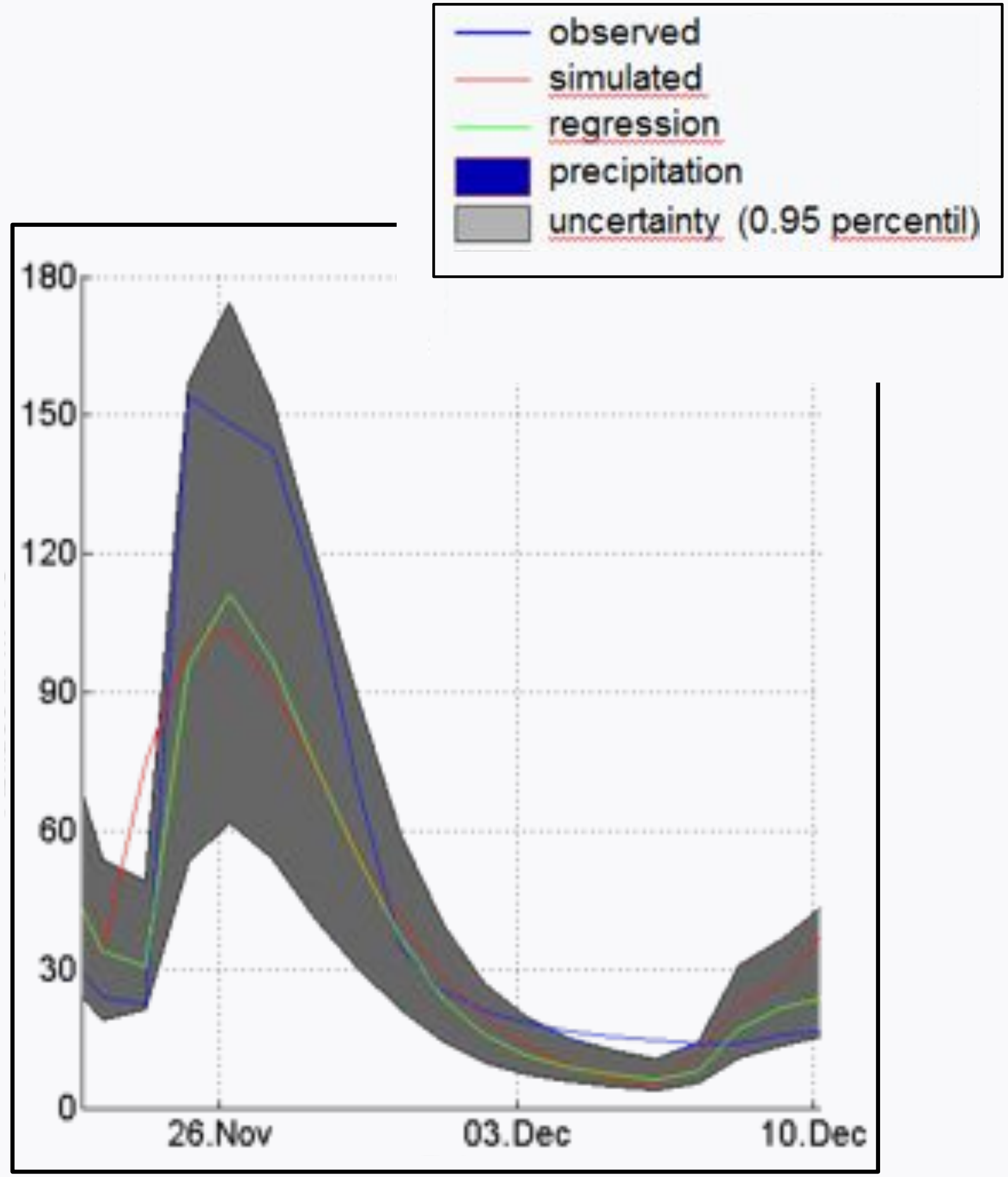
Final result – Event 2



Final result

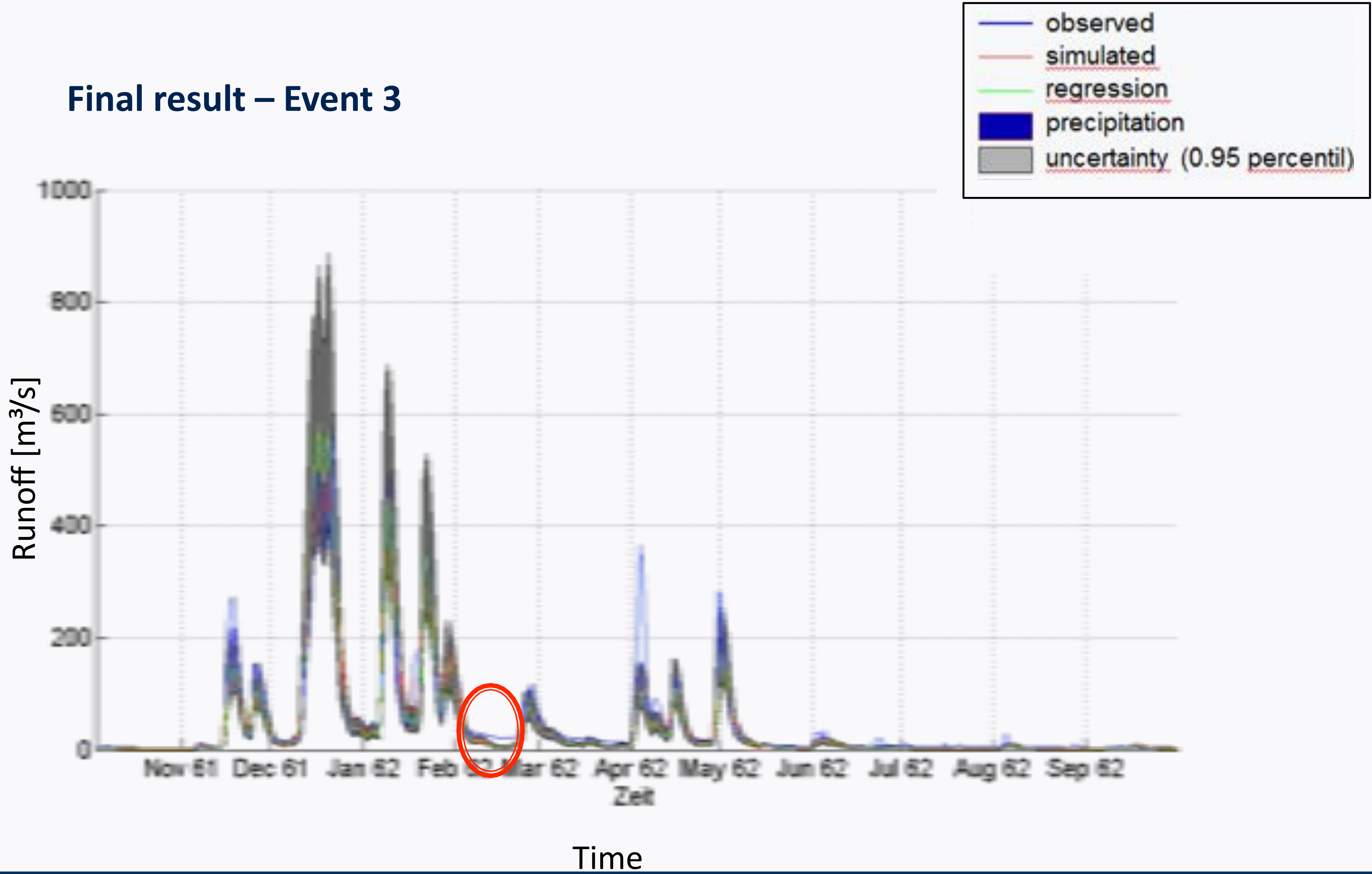
Final result – Event 2





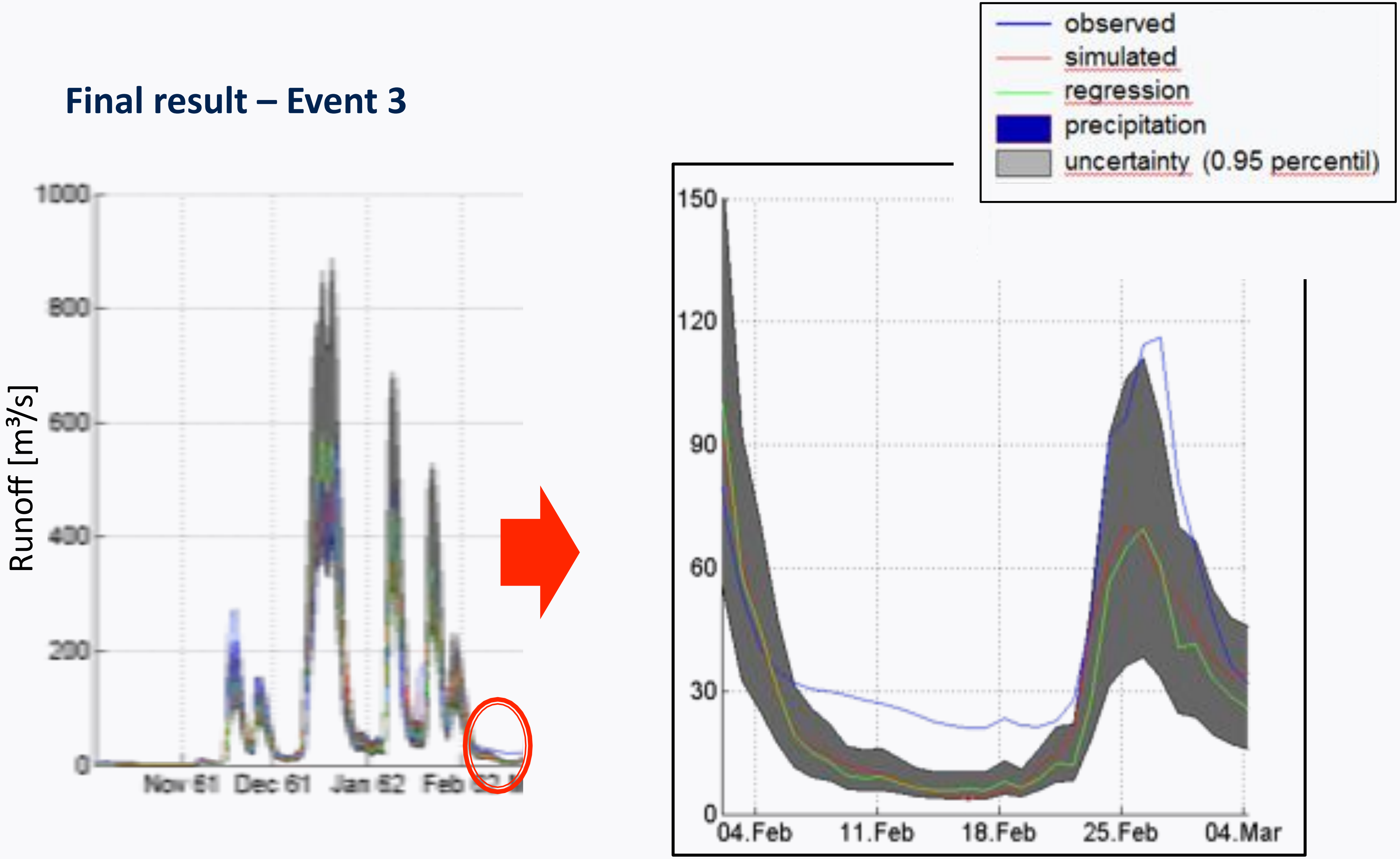
Final result

Final result – Event 3



Final result

Final result – Event 3



Summary

Limitations:

- long measured time series are required
- representation of non-stationary in practice very difficult
- extrapolation of the regression depends on the parameters of the Gaussian Process is modeled

Advantages:

- statistically based prediction of model error
- quality of resulting uncertainty bounds is not bad
- computational effort is relatively low

Conclusion

Gaussian Process Regression is an alternative approach to predict the error of hydrological models

Thank you for your attention!

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