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Stochastics in hydrometeorological processes: from point to global spatial scales and from minute to climatic time scales

Study on multifractal modeling of spatial rainfall

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ABSTRACT

The identification of the spatial structure of rainfall is widely recognized as a key issue in the hydrological applications. An approach to this problem is based on the empirical detection of some regularities in hydrological observations, such as the scale-invariance properties of rainfall. Scaling properties can provide simple relationships to link the statistical distribution of the rainfall process at different spatial and temporal scales, in the ranges of which the power-law assumption can be verified [1]. However, it is very difficult if not impossible to be able to properly capture the high spatial variability of rainfall fields with traditional rain gauge networks, while modern weather radars are, potentially, an instrument capable of meeting this need because of the fine spatial resolution of radar data.

This work focuses on the analysis of the scaling properties of rainfall in space by using data from a high density rain gauge network and from a weather radar both covering the urban area of Rome. The aim of the study is the identification of spatial scaling regimes, their ranges of validity, and the evaluation of the corresponding scaling properties.

1- SCALING PROPERTIES OF SPATIAL RAINFALL FIELDS

The measure *μ(λ)* at the scale *λ* of the rainfall field (positive and isotropic) $R(x_1,x_2)$, $x_1,x_2 \in [0,L]$

$$\mu(\lambda)=\frac{\lambda^2}{L^2}\int_{x_1}^{x_1+\delta l}\int_{x_2}^{x_2+\delta l} R(x_1,x_2)\,dx_2\,dx_1 \qquad (1)$$

where $\lambda = L/\delta l$ ($\lambda_1 = 1$ is the largest scale of interest) is used here instead of the length of the scale of homogeneity δl, defined for a set of size *L* embedded in a 2-dimensional space. The measure *μ(λ)* is said to be multiscaling (*multifractal*) if the *q*-moments of the *λ*-scale measure satisfy the condition

$$S_q(\lambda)=\left\langle[\varepsilon(\lambda)]^q\right\rangle \sim \lambda^{k(q)} \qquad (2)$$

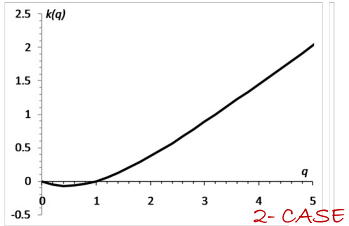
where [2]

$$\varepsilon(\lambda)=\frac{\mu(\lambda)}{\langle\mu(\lambda_1)\rangle} \qquad (3)$$

<·> = ensemble average

k(q) = exponent function, non-linear function of *q* (a linear behavior of the exponent *k(q)* would imply a simple scaling measure)

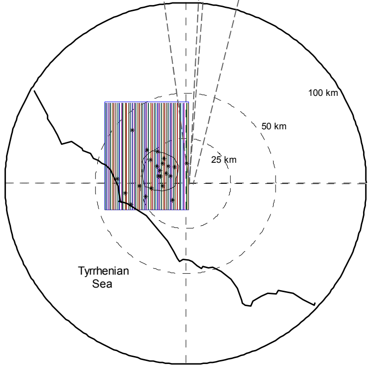
The multiscaling properties can be inferred and represented by analyzing the behaviour of the empirical moments scaling function *k(q)* estimated from the scaling of the *q*-moments of the non-dimensional rainfall intensity *ε(λ)* at different *λ*-scales.



2- CASE STUDY

The radar data come from the Polar 55C polarimetric C-band Doppler weather radar located in Rome, Italy. Radar measurements have a range-bin resolution of 75 m up to 120 km from the radar site. The temporal resolution is 5 minutes. After a transformation from Polar to Cartesian coordinates, a regular domain (47 km × 60 km) centred on the radar site is built. The finest resolution is 1 km^2.

The study area is covered by a high density rain gauge network which consists of 23 sites, and the gauge record at each site has 10-minute time resolution and about 18-year length (from July 1992 to December 2009).



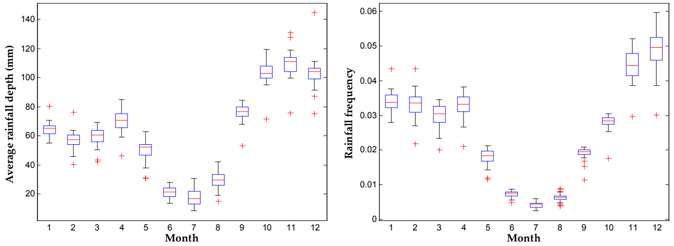
DAY	BEGINNING (UTC)	END (UTC)
15 APRIL 2008	07:10	17:20
24 APRIL 2008	16:05	18:00
15 SEPTEMBER 2008	13:30	23:59
3 OCTOBER 2008	00:00	23:59
28 OCTOBER 2008	00:00	23:59
4 NOVEMBER 2008	07:50	23:59
12-14 NOVEMBER 2008	00:00	23:55
24 NOVEMBER 2008	10:10	23:50
28 NOVEMBER 2008	07:45	23:55
29 NOVEMBER 2008	00:00	23:50
1 DECEMBER 2008	09:35	23:59
5 DECEMBER 2008	00:00	13:30
15 DECEMBER 2008	00:00	23:50

Radar dataset

Rain events recorded by the Polar 55C weather radar used in our work. Apart from the month no distinctions have been made among such events.

Summary statistics

Monthly series are considered to ensure temporal stationarity of the process and to highlight the seasonal effects.

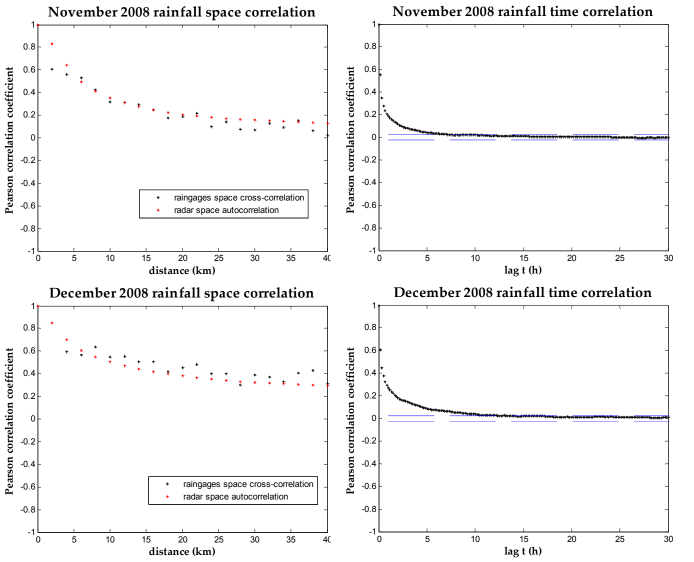


Box plots

The box has lines at the lower quartile, median, and upper quartile values.
Whiskers extend from each end of the box to the adjacent values in the data - the most extreme values within 1.5 times the interquartile range from the ends of the box.
Outliers are data with values beyond the ends of the whiskers. Outliers are displayed with a red + sign.

Correlation properties of rainfall fields

Correlation structure of rainfall data on the study area from the rain gauge network and the radar.


November 2008 rainfall space correlation

November 2008 rainfall time correlation

December 2008 rainfall space correlation

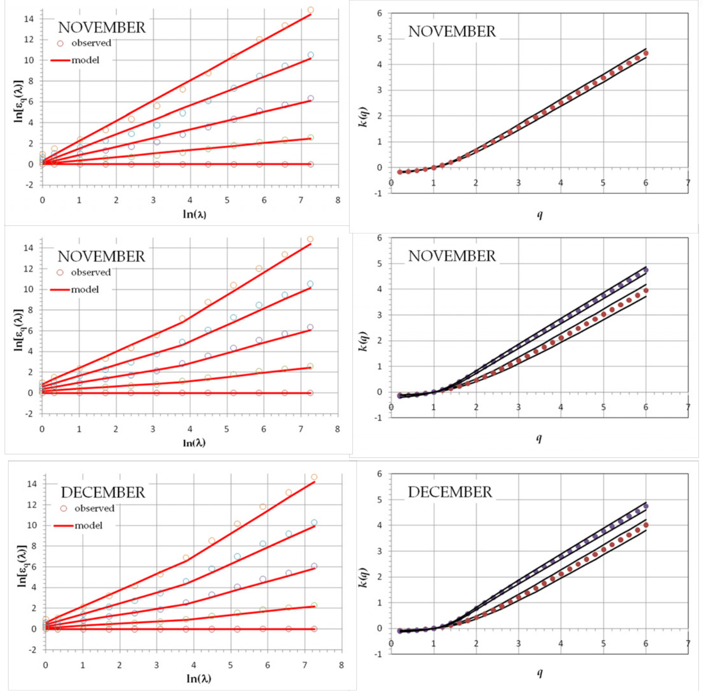
December 2008 rainfall time correlation

Multiscaling properties of radar rainfall fields

The estimation of *k(q)* can be performed by linear regression, when revealed, between the logarithm of the *q*-moments *lnS$_q$(λ)]* and the logarithm of the scale *ln(λ)* for different *q*. Because of the presence in the random field $R(x_1,x_2)$ of null values we consider only *q>0* (following equation (2)).

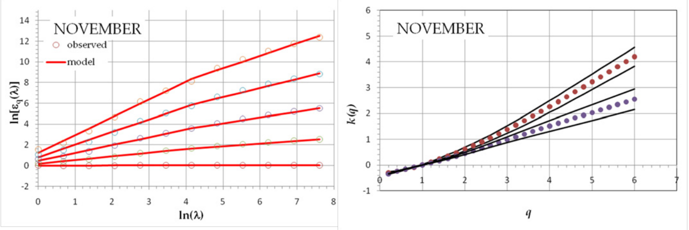
Statistical moments at different scales are computed over non-overlapping intervals of side δl ranging from *1 km* to about *40 km*. Results, showed for the two months of November and December 2008 in the graphs above, indicate the presence of breaks in the scaling whose position is about at δl *= 4 km* for both of the months. Furthermore, the *k(q)* trends with increasing *q* are showed for the two months. The 90% confidence intervals are also plotted.

Multiscaling properties of radar rainfall fields



Multiscaling properties of rainfall time series

Statistical moments at different time scales are also computed over non-overlapping intervals of side δt ranging from *10 min* to *14 d*. Results are the average behaviour of *q*-moments of the different gages over the study area in the entire observation time. Similar breaks in the scaling are detected which do not vary among different months (the scaling break detected is at about *5 h*) .



CONCLUSIONS

This preliminary analysis shows the existence of space scaling properties for the radar rainfall events examined. Furthermore, radar rainfall space correlation structure is in good agreement with the space cross-correlation estimated by means of the high density rain gauge network over the study area. In analyzing scaling properties, scaling breaks are detected both in the space and in the time domains. Future works need to increase the radar dataset and to further analyze scaling properties with the codimension function of singularities [3].

REFERENCES

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[3] Lavallée, D., Lovejoy, S. and Schertzer, D. (1991) On the determination of the codimension function, in Non-linear variability in geophysics: Scaling and Fractals, edited by D. Schertzer, and S. Lovejoy, pp. 99-110, Kluwer.