3rd STAHY International Workshop on Statistical Methods for Hydrology and Water Resources Management.

October 1-2, 2012, Tunis, Tunisia.

A Monte Carlo procedure based on the use of copulas for risk assessment of dam overtopping.

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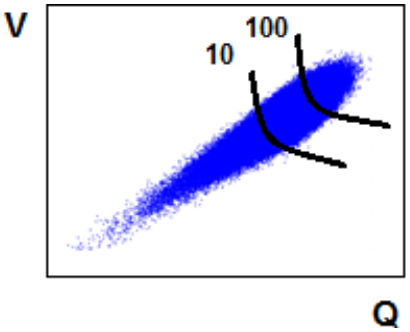
1. INTRODUCTION

- **A flood event can be characterised by the peak, volume and duration.**

- **Multivariate frequency analysis → to study the probability of occurrence of a flood event.**

- **Joint return period curves are easily obtained by means of a copula model.**

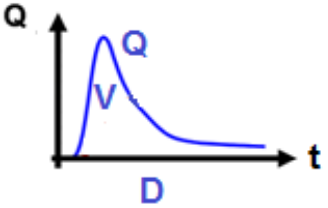


- **However, the return period is the time elapsed between two successive events exceeding a threshold.**

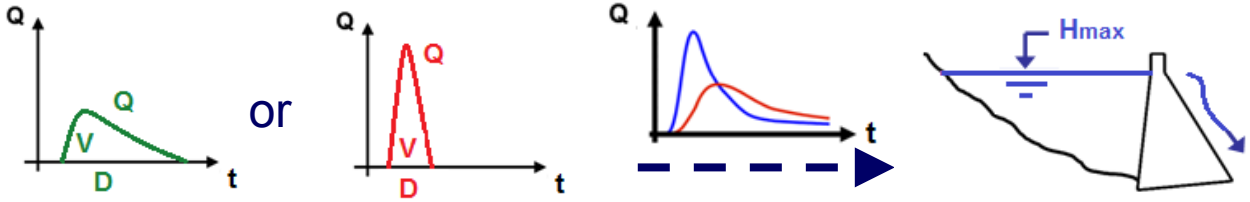
- **As the hydrological risk to the dam is related to the maximum water level reached → that threshold needs to be defined in terms of maximum water elevation.**

1. INTRODUCTION

- **Maximum water level reached depends on:**
  - **Hydrograph**: peak, volume, duration and shape.
  - **Dam**: spillway length, flood control volume and operating rules.

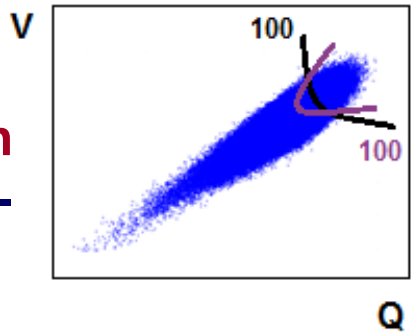


- **Different events can reach the same water level.**

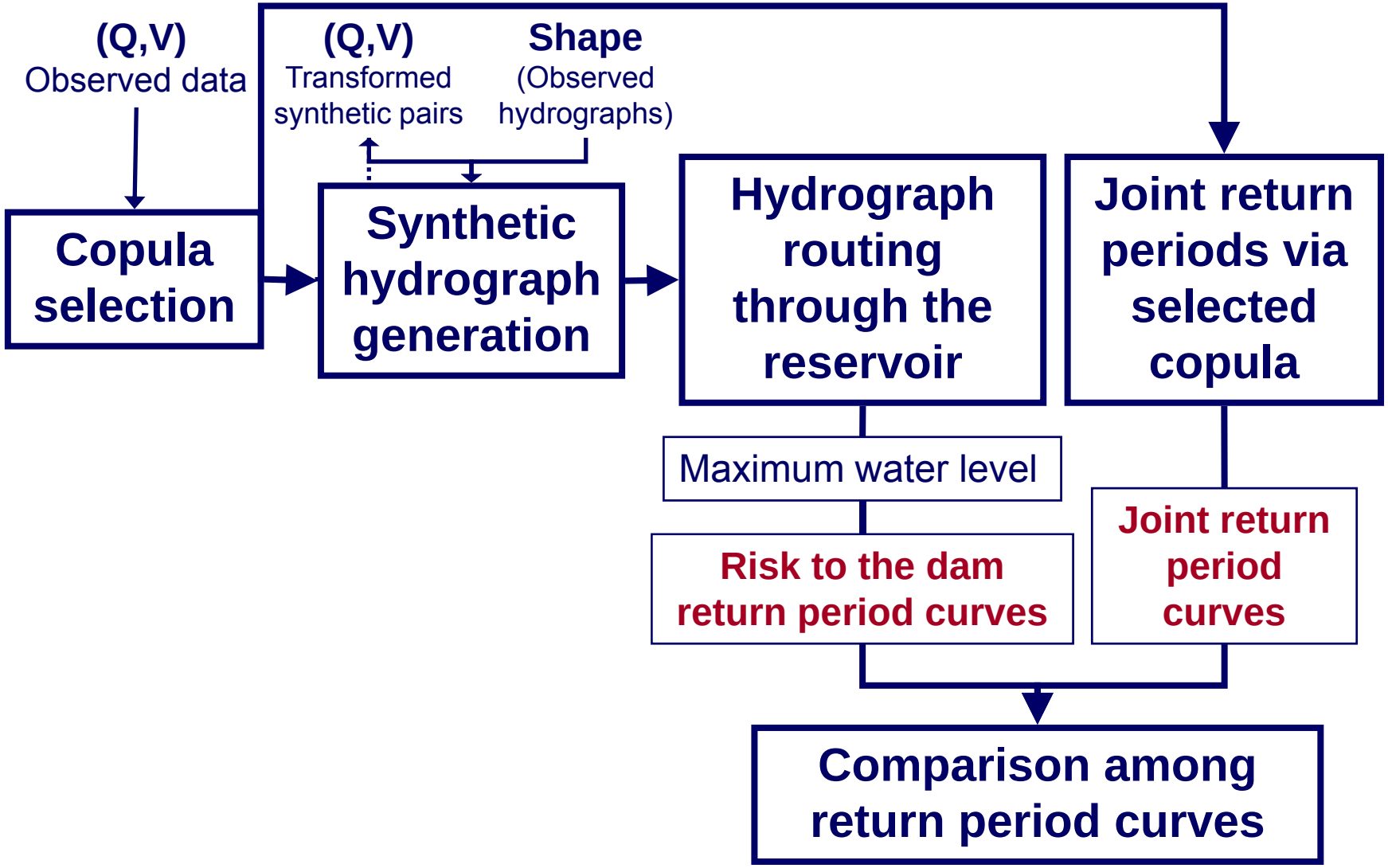


- **These events will have the same risk to the dam and will be in the same return period curve.**

- **In the present study both kinds of return period curves are analysed in the peak-volume space.**



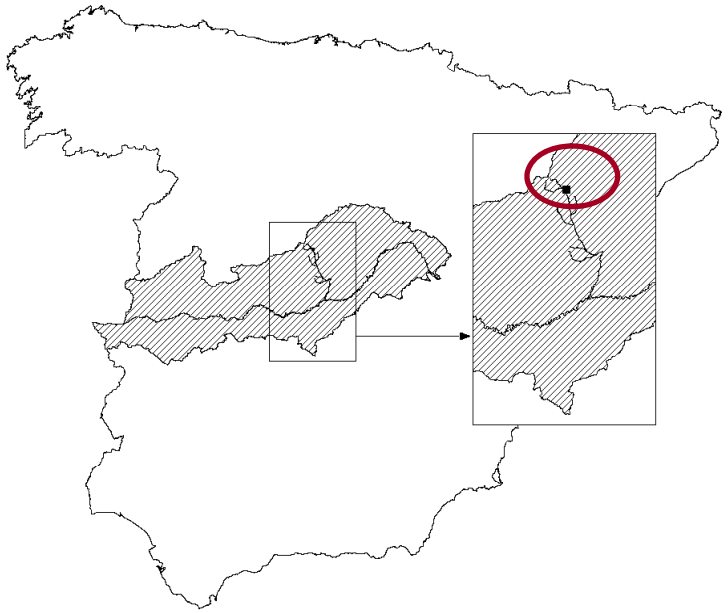
2. METHODOLOGY



A Monte Carlo procedure based on the use of copulas for risk assessment of dam overtopping.

3. CASE STUDY

❑ Santillana Reservoir

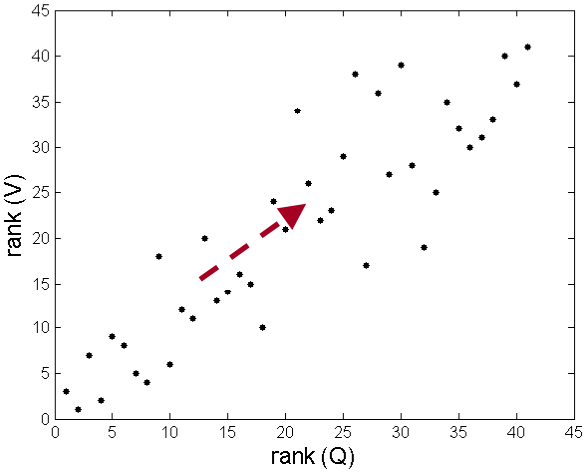


- **Uncontrolled spillway.**
- **Drainage area: 325.6 km^2.**
- **Reservoir volume: 92 hm^3.**
- **Spillway capacity: 300 m^3/s.**
- **Observed data:**
  - Maximum annual flood peak (Q) in m^3/s.
  - Flood volume (V) in hm^3.
- **Univariate marginal distributions:**
  - Gumbel distribution (L-moments method).

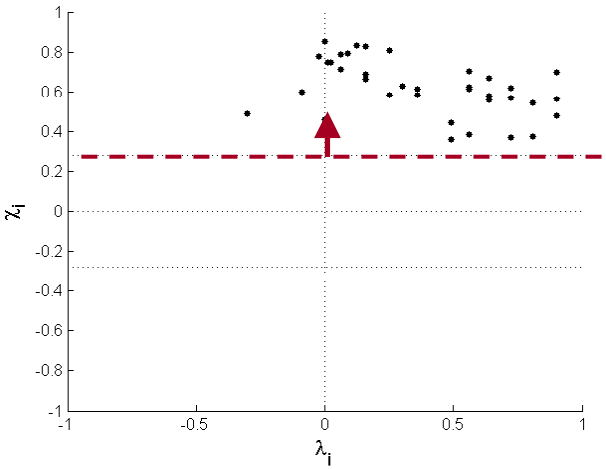
COPULA SELECTION

❑ Dependence evaluation → Positive association

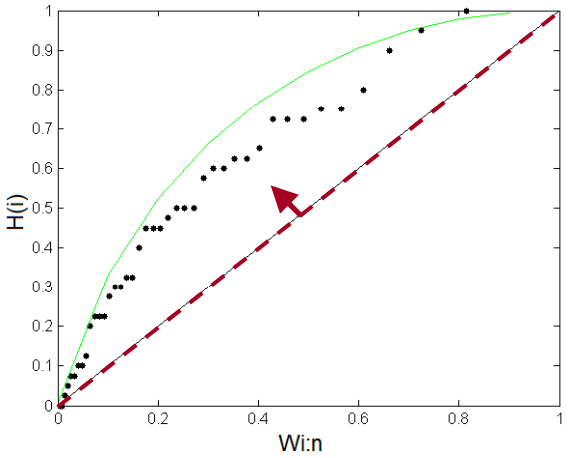
- **Graphical analysis**



Scatter plot



Chi-plot



K-plot

- **Non-parametric rank based dependence measures**

Measure	value	p-value
Kendall's tau	0.7244	2.53e-11
Spearman's rho	0.8899	1.82e-08

COPULA SELECTION

❑ Copulas parameter estimation

- **Copulas firstly considered:**

~~AMH~~ | Clayton | Frank | Gumbel | Galambos

Hüsler-Reiss | ~~Tawn~~ | ~~FGM~~ | Plackett

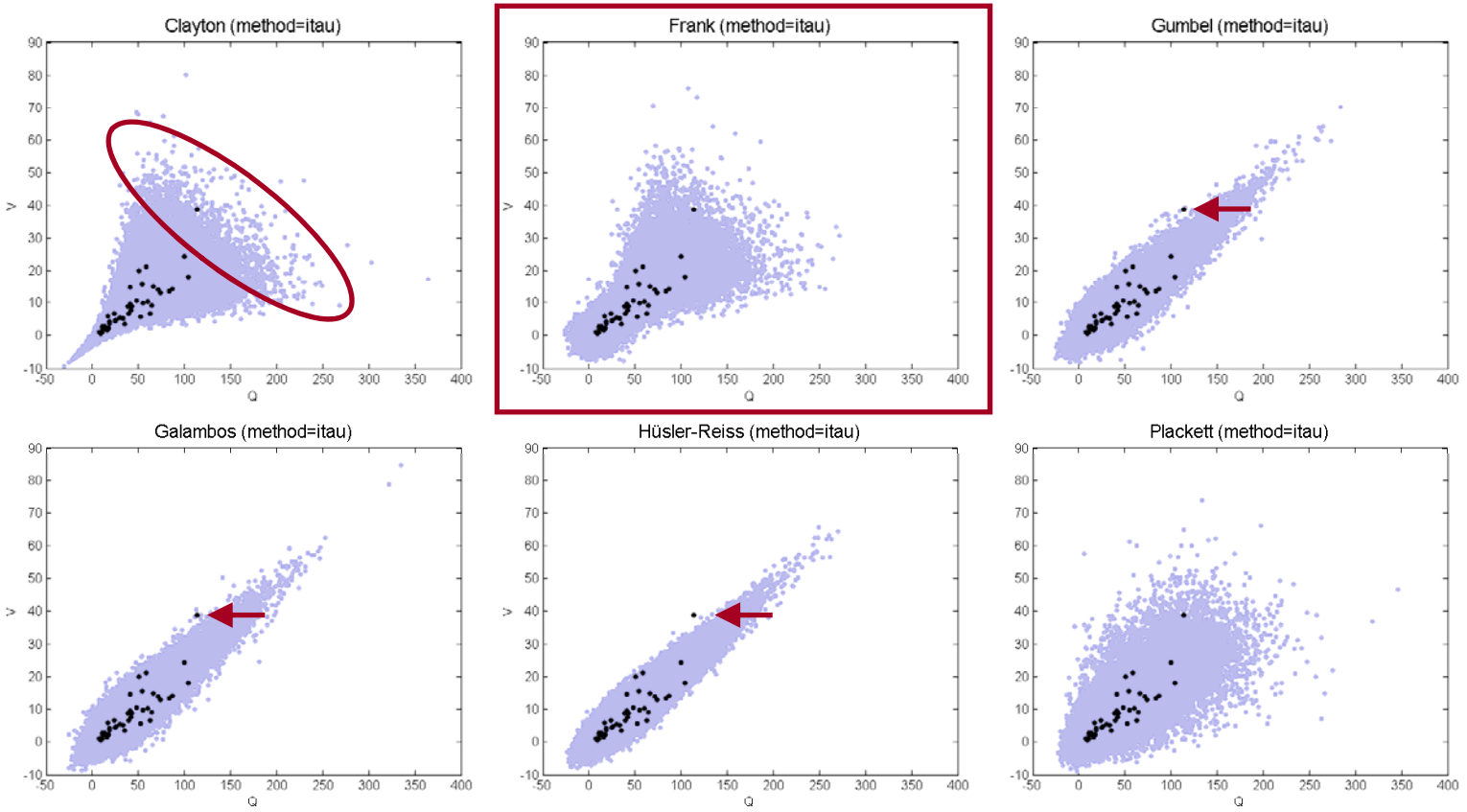
- **Eliminated copulas via Kendall's tau: AMH, Tawn and FGM.**
- **Rank based parameter estimation methods:**
  - ➢ **Inversion of Kendall's tau method.**
  - ➢ **Maximum pseudo-likelihood method.**

❑ Generation of synthetic pairs: 100,000 (U_1,U_2)

COPULA SELECTION

❑ Goodness-of-fit test

- **Graphical tools: observed data vs. generated sample (Q,V).**



COPULA SELECTION

❑ Goodness-of-fit test

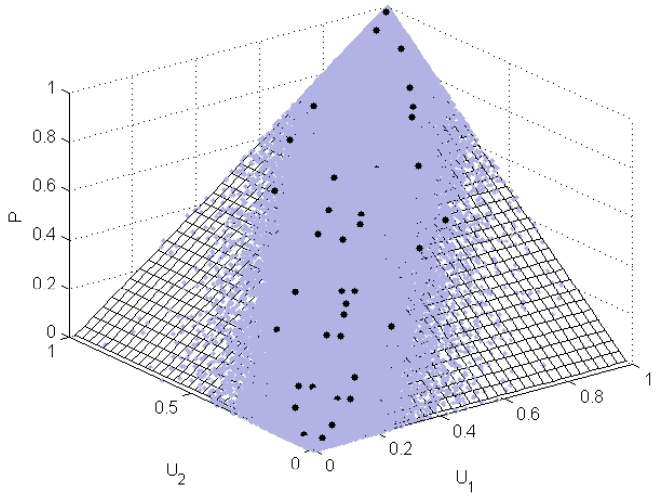
- **Formal test:**
  - **Cramér von-Mises statistic (S_n) based on the empirical copula.**
  - **P-value based on 10,000 parametric bootstrap samples.**

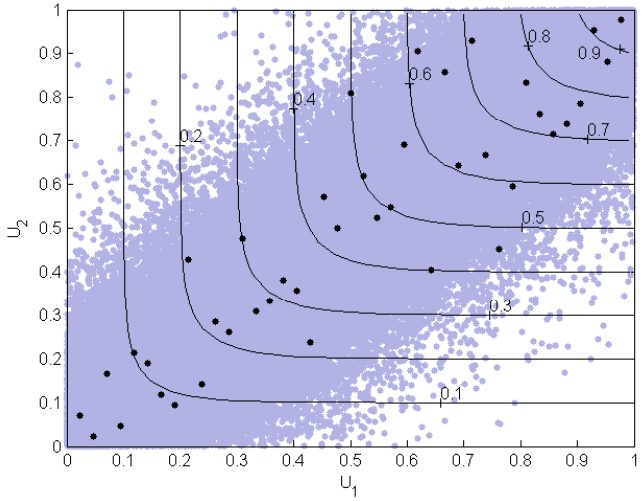
Copula	Parameter estimation method	θ	St. Error	S_n	P-value
Clayton	Inversion Kendall's tau	5.257	1.202	0.0223	0.3828
	MPL	3.337	1.081	0.0524	0.0565
Frank	Inversion Kendall's tau	12.622	2.459	0.0174	0.8402
	MPL	11.774	2.858	0.0202	0.7332
Gumbel	Inversion Kendall's tau	3.628	0.601	0.0218	0.3967
	MPL	3.068	0.714	0.0351	0.0649
Galambos	Inversion Kendall's tau	2.919	0.602	0.0219	0.3910
	MPL	2.345	0.697	0.0357	0.0603
Hüsler-Reiss	Inversion Kendall's tau	3.677	0.684	0.0221	0.3663
	MPL	2.970	0.777	0.0379	0.0568
Plackett	Inversion Kendall's tau	54.230	21.699	0.0181	0.7893
	MPL	33.570	17.531	0.0308	0.2967

COPULA SELECTION

❑ Selected copula

- **Frank copula with its parameter obtained by inversion of Kendall's tau method.**

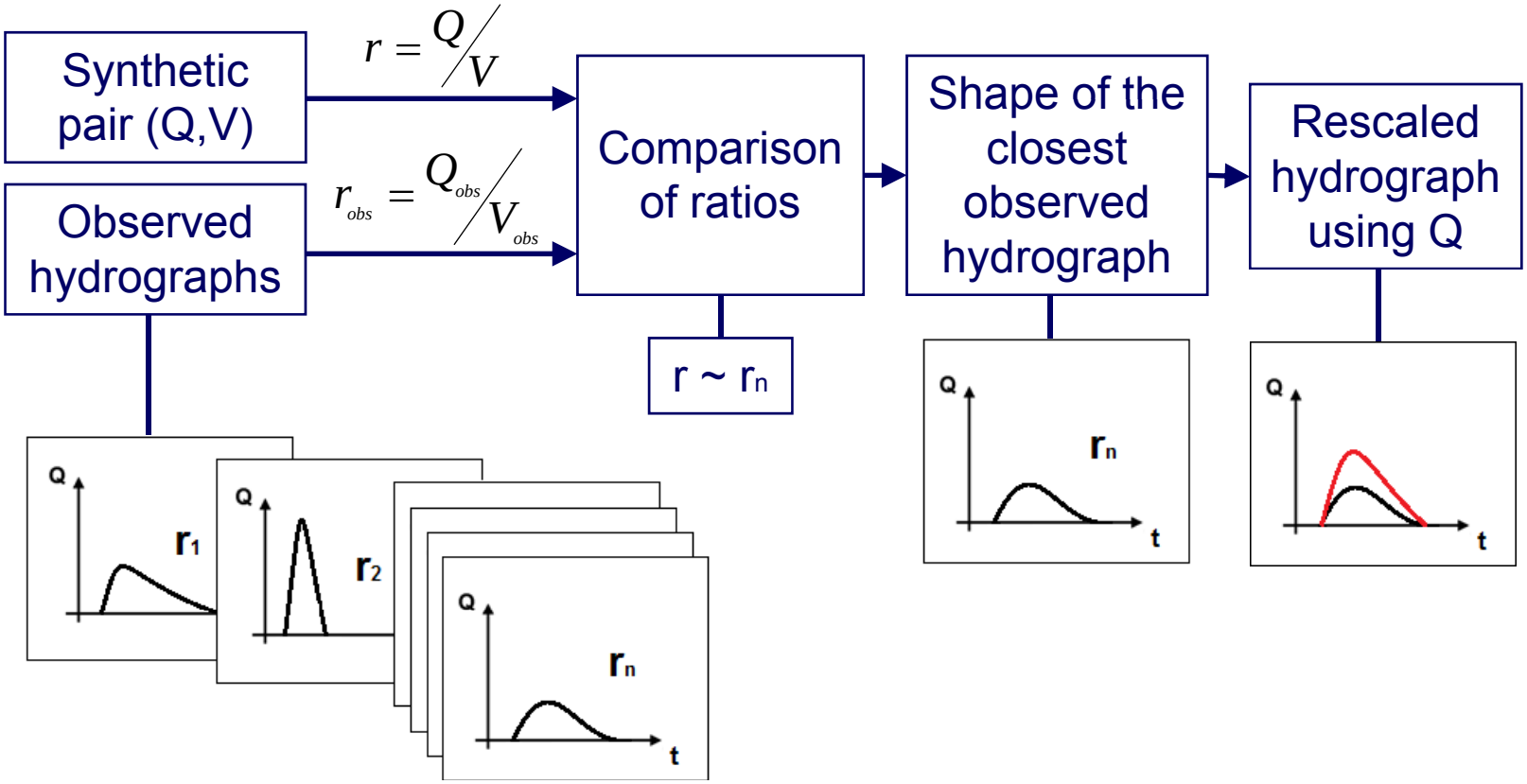




RESULTS

❑ Synthetic hydrograph generation

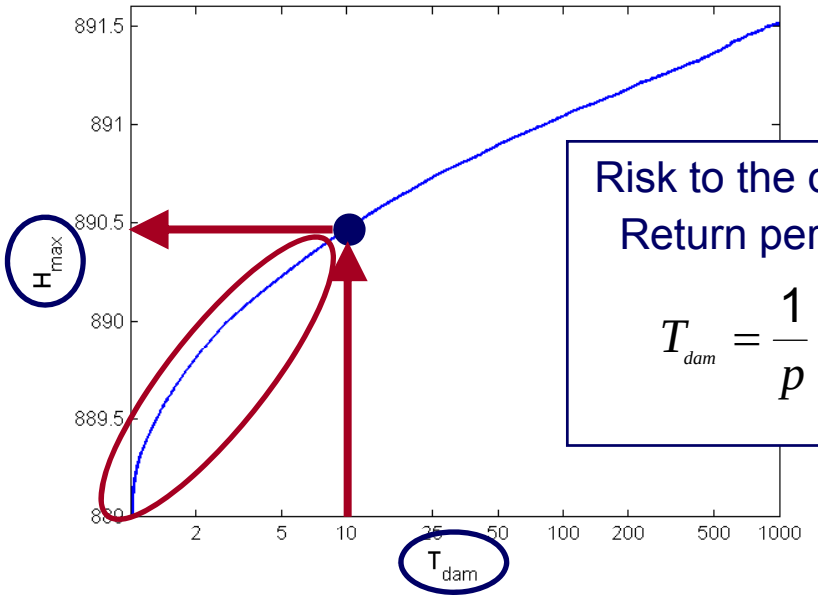
- **100,000 synthetic pairs (Q,V).**



RESULTS

❑ Hydrograph routing through the reservoir

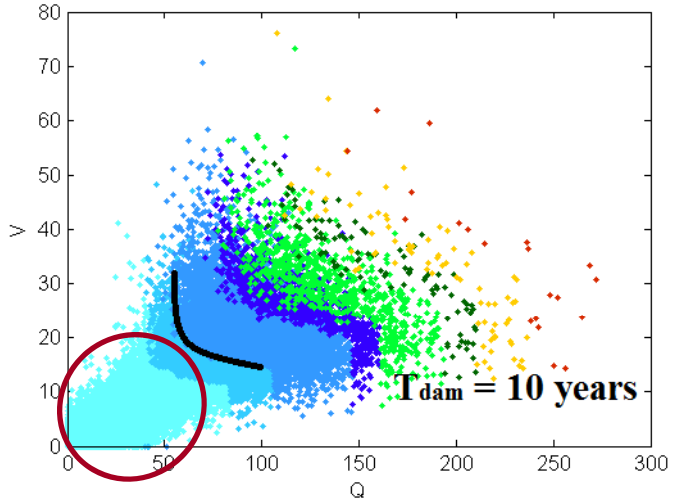
Frequency curve of the maximum water level reached



Risk to the dam: Return period

$$T_{dam} = \frac{1}{p}$$

Risk to the dam: Return period curves

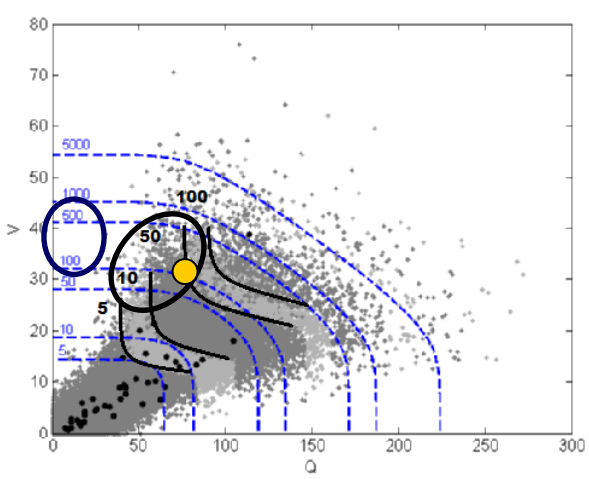


p = probability of exceeding a water level.

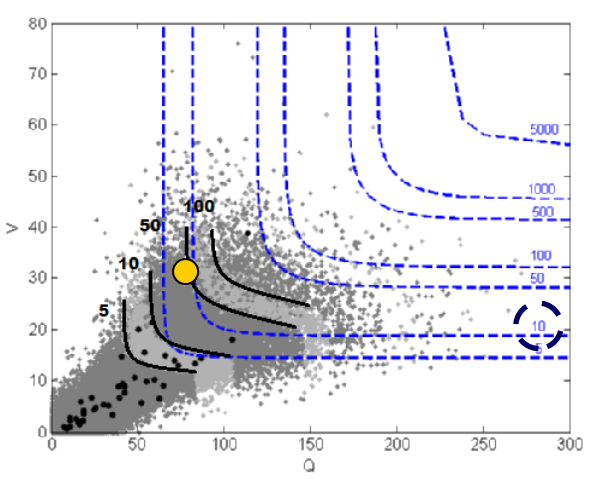
RESULTS

❑ Joint return period curves

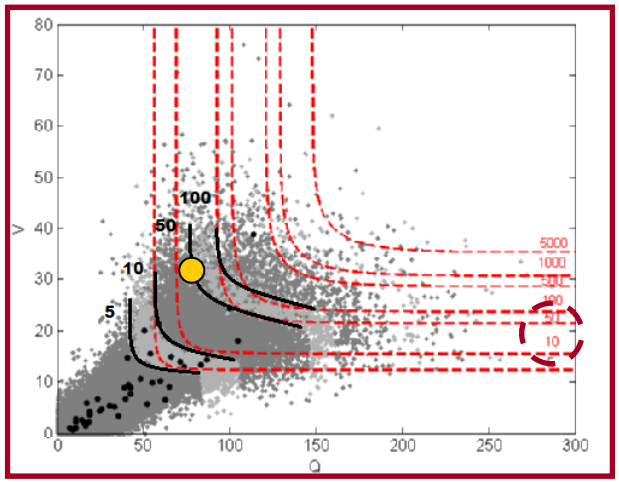
- **Via Frank copula: $T_{xy}^{\wedge}, T_{xy}^{\vee}$ (primary) and $\rho_t^{\vee}$ (secondary).**



$T_{xy}^{\wedge}, T_{dam}$



$T_{xy}^{\vee}, T_{dam}$



$\rho_t^{\vee}, T_{dam}$

Q (m^3/s)	V (hm^3)	t	$T_{x,y}^{\vee}$	T_x	T_y	$T_{x,y}^{\wedge}$	$K_{\theta_n}(t)$	$\rho_t^{\vee}$	m.w.l.	T_{dam}
82.32	31.75	0.9	10	10	94	137	0.9568	23	890.79	31

5. CONCLUSIONS

- A methodology to compare the probability of occurrence of a flood with the return period linked to the risk at the dam was developed.
- Synthetic hydrographs were generated by a Frank copula and were routed through the reservoir.
- The maximum water level reached at the dam was used as a surrogate of its hydrological risk.
- Instead of using the secondary return period, flood hydrographs should be routed to estimate the risk to the dam.
- The proposed methodology procures useful information in order to estimate the Design Flood Hydrograph.
- Future work → evaluate the sensitivity to different reservoir volumes and spillway lengths.

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Thanks for your attention