Joint Return Periods in Hydrology:
a practical review focussing on synthetic design hydrograph estimation.

Joint Return Period Estimation?

in hydraulics, hydrology: notion of return period to quantify design parameters

1D: well known methods

moreD: several challenges
- dependence between design variables?
- definition of joint return period (JRP)?
- how to apply?

Case study: design hydrograph estimation

Aim: calculate design values for Qp and V for JRP = 100 years

different JRP definitions (in 2D and 3D)

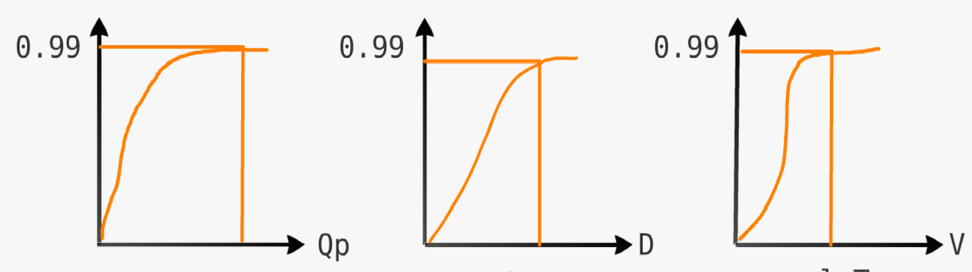
evaluate differences

UNIVARIATE DESIGN QUANTILES

= 1 for annual maxima

$$T = \frac{\mu_T}{1-F_X(x_T)} \Leftrightarrow x_T = F_X^{-1}\left(1-\frac{\mu_T}{T}\right)$$

= 0.99 for T = 100

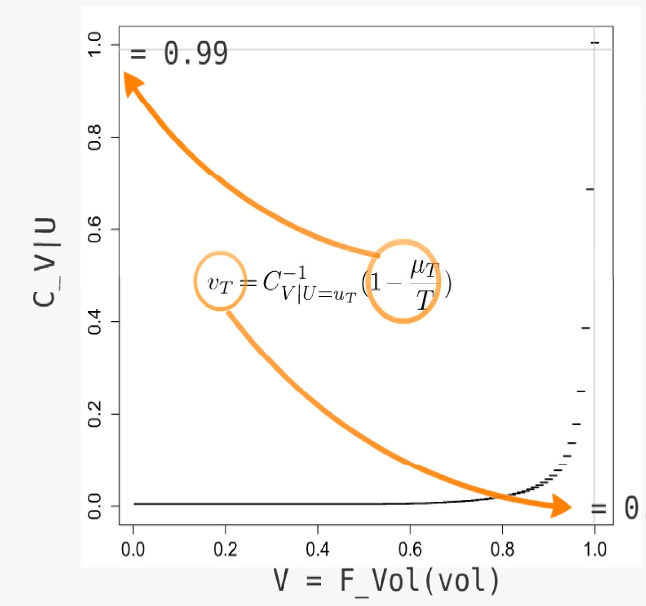


qp_T = 303 m³/s   d_T = 28.4 m³   vol_T = 4.42 10^6 m³

CONDITIONAL COPULA DESIGN QUANTILES

$$T = \frac{\mu_T}{1-C_{V|U=u_T}(v_T)}$$

conditional CDF of Vol for univariate quantile of Qp, expressed by 2D copula

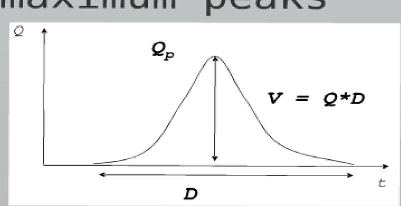


through inverse CDF of V: vol_T = 6.13 10^6 m³

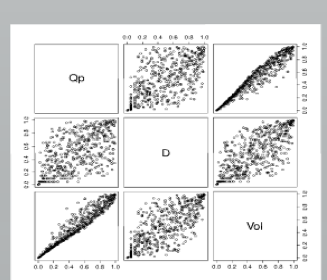
Discharge case study

500 simulated annual maximum peaks

Qp : peak discharge
D : duration
Vol : volume



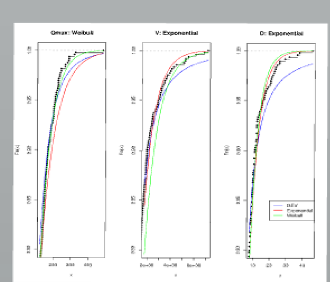
dependence



Kendall's tau

(Qp,D) -> 0.42
(Qp,Vol) -> 0.85
(Vol,D) -> 0.54

marginal distribution functions



fit in the tails is most important

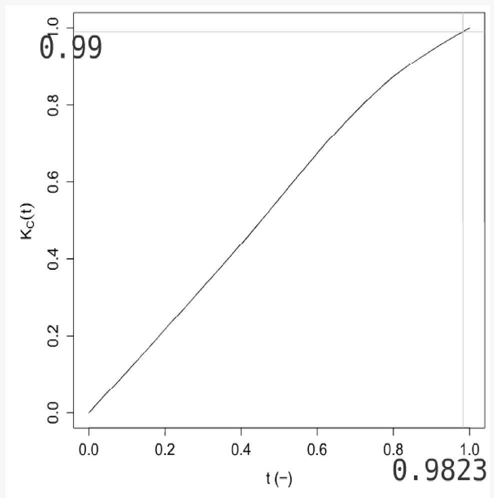
2D KENDALL DESIGN QUANTILES

Kc-function = CDF of copula values t

theoretically sound extension of univariate return period definition: sub-critical and super-critical events

$$T = \frac{\mu_T}{1-K_C(t)}$$

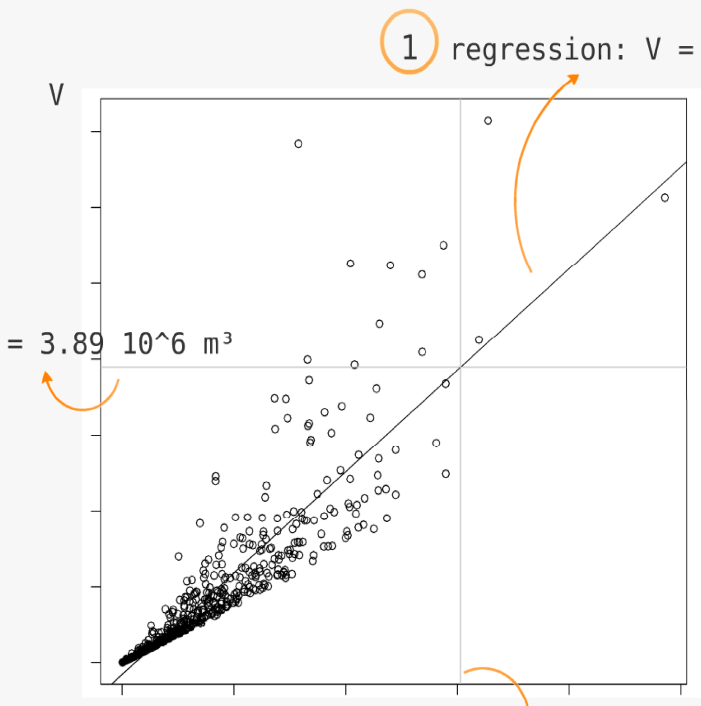
$$t = K_C^{-1}\left(1-\frac{\mu_T}{T}\right)$$



t-level is found, select point with highest joint likelihood:

(u_T,v_T) = (0.9871, 0.9870)
(qp_T,vol_T) = (290 m³/s, 4.16 10^6 m³)

REGRESSION DESIGN QUANTILES



Which approach to choose?

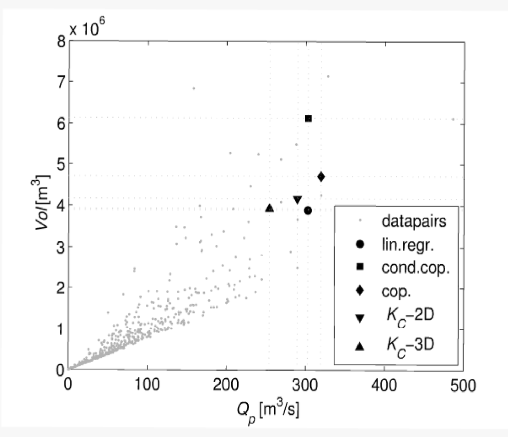
Choice of JRP method influences design event

incorporation of dependence better by copula than by regression

methods based on Kendall's function are mathematically consistent with univariate theory

importance of a good copula fit

vine-copulas seem promising for flexible multivariate copula constructions



Future research efforts

joint efforts on joint return periods

ensemble generation

vine-copulas

comprehensive framework + guidelines for practitioners

Thanks for your attention on this late Friday afternoon!

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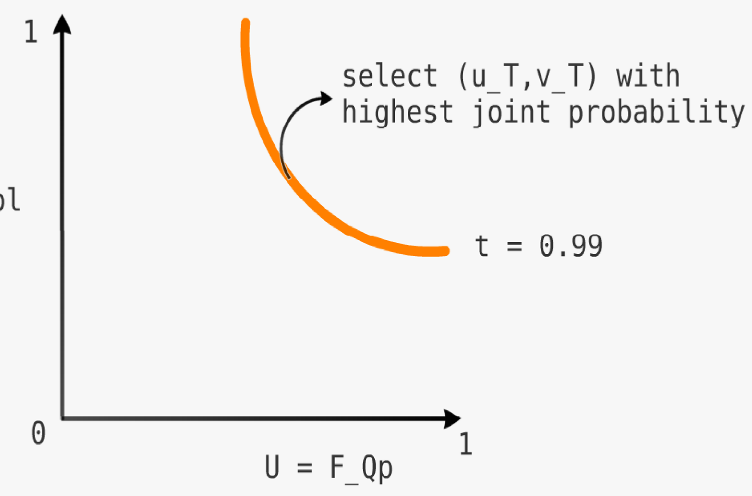
for more copula applications in hydrology, please visit website of

"International Commission on Statistics in Hydrology"

www.stahy.org

COPULA DESIGN QUANTILES

$$T = \frac{\mu_T}{1-F_{XY}(x,y)} = \frac{\mu_T}{1-C_{UV}(F_X(x),F_Y(y))} = \frac{\mu_T}{1-C_{UV}(u,v)}$$



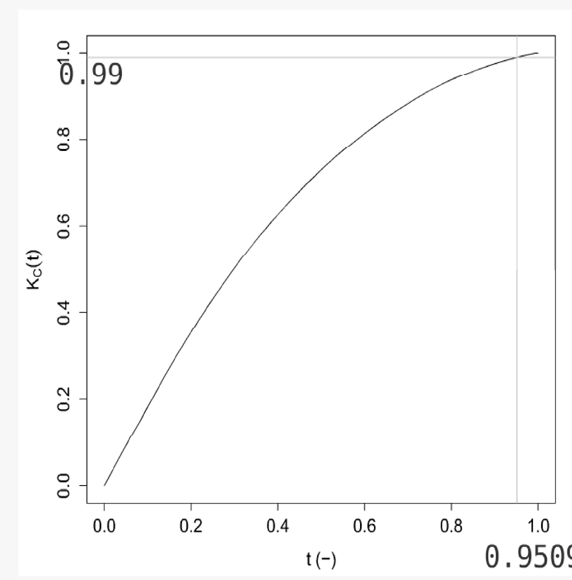
(u_T,v_T) = (0.9927, 0.9926)
(qp_T,vol_T) = (319 m³/s, 4.71 10^6 m³)

3D KENDALL DESIGN QUANTILES

3D vine-copula of (Qp,D,Vol) nummerically derived Kc-function

$$T = \frac{\mu_T}{1-K_C(t)}$$

$$t = K_C^{-1}\left(1-\frac{\mu_T}{T}\right)$$



t-level is found, select point with highest joint likelihood:

(u_T,v_T) = (0.9752, 0.9834)
(qp_T,vol_T) = (254 m³/s, 3.93 10^6 m³)

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